Lecture 8: More on Operational Amplifiers (Op Amps)

Input Impedance of Op Amps and Op Amps Using Negative Feedback:

- Consider a general feedback circuit as shown.
  - Assume that the amplifier has input impedance $R_{in}$.
  - We wish to find the input impedance $R'_\text{in}$ of the circuit including the effect of negative feedback.
  - For the case of no feedback ($B = 0$) we have:
    \[
    R_{\text{in}} = \frac{V_{\text{in}}}{I_{\text{in}}}
    \]
    \[
    I_{\text{in}} = \frac{V_{\text{in}}}{R_{\text{in}}}
    \]
  - If we include negative feedback (with $B < 0$) the input to the amplifier is:
    \[
    V_{\text{in}} + BV_{\text{out}}
    \]
  - The input current is now:
    \[
    I_{\text{in}} = \frac{(V_{\text{in}} + BV_{\text{out}})}{R_{\text{in}}}
    \]
  - We showed last week for a circuit with negative feedback:
    \[
    V_{\text{out}} = AV_{\text{in}}/(1 - AB)
    \]
    \[
    I_{\text{in}} = \frac{V_{\text{in}} + \frac{ABV_{\text{in}}}{1 - AB}}{R_{\text{in}}}
    \]
    \[
    = \frac{V_{\text{in}}}{R_{\text{in}} (1 - AB)}
    \]
    \[
    = \frac{V_{\text{in}}}{R'_{\text{in}}}
    \]
    \[
    R'_{\text{in}} = R_{\text{in}} (1 - AB)
    \]

Input impedance with negative feedback is much larger than the no feedback case. It is also possible to lower $R'_{\text{in}}$ with negative feedback.
● Input impedance of non-inverting amplifier:
  - The input voltage is directly connected to the op amp
    - the input impedance is expected to be large.
  - The typical input resistance of a 741 op amp is 2 MΩ (no feedback case).
  - Pick $R_1 = 1 \, k\Omega$ and $R_f = 50 \, k\Omega$
    - amplifier gain $G \sim R_f / R_1 = 50$
    - $B = 1/G = 0.02$
  - The open loop gain ($A$) as a function of frequency for the 741 can be read off the spec sheets.
    - Calculate the input impedance of the non-inverting amp vs. frequency:

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>$A$</th>
<th>Input Impedance $R'_\text{in}$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^1$</td>
<td>$10^5$</td>
<td>$4 \times 10^9$</td>
</tr>
<tr>
<td>$10^3$</td>
<td>$10^3$</td>
<td>$4 \times 10^7$</td>
</tr>
<tr>
<td>$10^6$</td>
<td>1</td>
<td>$2 \times 10^6$ (R of op amp)</td>
</tr>
</tbody>
</table>

● Input impedance of inverting amplifier:
  - Point A is at ground (a virtual ground)
    - The input voltage does not actually "see" the op amp.
    - The input impedance of this configuration is simply:
      $$R'_\text{in} = V_{\text{in}} / I_{\text{in}} = R_1$$
  - If we use the same resistors as in the non-inverting amplifier ($R_1 = 1 \, k\Omega$ and $R_f = 50 \, k\Omega$)
    - the input impedance of this amp is 1 kΩ, independent of frequency.

  Thus the inverting amp has a low input impedance.
  - This is one of the practical drawbacks to this amplifier configuration.
Output Impedance of Op Amps Using Negative Feedback:

- The output impedance of a circuit is defined as:
  \[ R_{\text{out}} = \frac{V_{\text{out}}}{I_{\text{out}}} \]

- We wish to see how the above expression is modified by negative feedback.
  - Assume \( V_{\text{in}} \) is grounded.
  - Assume we put a voltage \( V \) at the output of the amp.
    - The feedback network puts \( BV_{\text{out}} \) \((B < 0)\) back to the input.
    - This voltage appears across the input impedance as \( V_d \).

\[ I_{\text{out}} = \frac{V_{\text{out}} - AV_d}{R_{\text{out}}} \]
\[ = \frac{V_{\text{out}} - ABV_{\text{out}}}{R_{\text{out}}} \]
\[ = \frac{V_{\text{out}} (1 - AB)}{R_{\text{out}}} \]
\[ = \frac{V_{\text{out}}}{R'_{\text{out}}} \]

- The new output impedance is greatly reduced:
  \[ R'_{\text{out}} = \frac{R_{\text{out}}}{1 - AB} \]
  - \( R'_{\text{out}} \rightarrow 0 \) as \( A \rightarrow \infty \).
Op Amp Stability and Compensation

- A major reason for using negative feedback with op amps is to make the amp stable against oscillations.
  - It is still possible to drive the amp into oscillation under certain conditions.
  - From a previous lecture we derived the gain equation for amps with feedback:
    \[ G = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{A}{1 - AB} \]
    - Oscillations occur when \( AB \to 1 \).
    - This can occur for positive feedback.
  - In principle, the inverting input of the op amp adds a fraction (determined by the feedback network) of the output to the input with a relative phase of \( 180^0 \).
  - However at high frequencies this phase shift decreases, eventually reaches zero the circuit can become unstable (i.e. oscillate).
  - Since the op amp is made up of many resistors and capacitors we can model these phase shifts using RC networks.
  - Recall for a low pass RC filter the gain and phase shift is given by:
    \[ G = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \]
    \[ \tan \phi = -\omega RC \]
    - At frequencies above the break point \( (\omega RC = 1) \) the gain falls off as \( 1/\omega \).
    - This falls off is 20 dB for each factor of 10 (or 6 dB per octave) increase in the frequency.
    - The phase shift rapidly converges to \(-\pi/2 \) or \(-90^0 \).
    - The phase shift that we want to avoid is \( 180^0 \).
    - In terms of voltage gain a filter that has the gain falling off as \( 1/\omega^2 \) will produce a \( 180^0 \) phase shift.
The easiest way to visualize this problem is by imagining two low pass RC filters in series since the gains of filters are multiplicative (but additive in dBs).

For 20 and 40 dB lines the frequency (x axis) at which the lines hit the gain curve is where \( A = -1/B \).
- If the phase shift at this frequency is \( 180^\circ \) oscillations will occur.

For the 40 dB line
- no oscillations can occur
- the gain rolloff is only 20 dB/decade.
  - the phase shift \( \leq 90^\circ \)

For the 20 dB line
- oscillations can occur
- the gain rolloff is 40 dB/decade
  - a \( 180^\circ \) phase shift is possible
Compensation:
- To make an op amp stable against oscillation
  - make sure the open loop gain \((A)\) falls off no faster than 20 dB/decade
  - not possible to have a 180° phase shift.
- Some op amps (e.g. \(\mu\)A741) are **internally compensated** (with capacitors) to insure that the gain roll-off is 20 dB or smaller all the way down to voltage gains of unity.
- A second type of op amp is called **uncompensated**
  - user adds compensating capacitors external to the op amp for stability against oscillation.
  - advantage: achieve higher gain by a suitable choice of capacitors
  - disadvantage: the circuit will oscillate if the wrong capacitor(s) was chosen!