Lecture 7: Transistors and Amplifiers

Hybrid Transistor Model for small AC:

- The previous model for a transistor used one parameter ($\beta$, the current gain) to describe the transistor.
  - doesn't explain many features of three common forms of transistor amplifiers (common emitter etc.)
  - e.g. could not calculate the output impedance of the common emitter amp.

- Very often in electronics we describe complex circuits in terms of an equivalent circuit or model.
  - need a model that relates the input currents and voltages to the output currents and voltages.
  - the model needs to be linear in the currents and voltages.
    - For a transistor this condition of linearity is true for small signals.

- The most general linear model of the transistor is a 4-terminal "black box".

\[ \begin{array}{c}
I_i \\
V_i \\
\hline
T \\
\hline
V_o \\
I_o
\end{array} \]

- In this model we assume the transistor is biased on properly and do not show the biasing circuit.
- Since a transistor has only 3 legs, one of the terminals is common between the input and output.
- There are 4 variables in the problem, $I_i$, $V_i$, $I_o$, and $V_o$.
  - The subscript i refer to the input side while the subscript o refers to the output side.
  - We assume that we know $I_i$ and $V_o$. 

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Kirchhoff’s laws relate all the currents and voltages:

\[ V_i = V_i(I_1, V_o) \]
\[ I_o = I_o(I_1, V_o) \]

For a linear model of the transistor with a small changes in \( I_1 \) and \( V_o \):

\[ dV_i = \left( \frac{\partial V_i}{\partial I_1} \right)_{V_o} dI_1 + \left( \frac{\partial V_i}{\partial V_o} \right)_{I_1} dV_o \]
\[ dI_o = \left( \frac{\partial I_o}{\partial I_1} \right)_{V_o} dI_1 + \left( \frac{\partial I_o}{\partial V_o} \right)_{I_1} dV_o \]

The partial derivatives are called the hybrid (or \( h \)) parameters:

\[ dV_i = h_{ii} dI_1 + h_{io} dV_o \]
\[ dI_o = h_{oi} dI_1 + h_{oo} dV_o \]

- \( h_{oi} \) and \( h_{io} \) are unitless
- \( h_{oo} \) has units \( 1/\Omega \) (mhos)
- \( h_{ii} \) has units \( \Omega \)

The four \( h \) parameters are easily measured.
- e.g. to measure \( h_{ii} \) hold \( V_o \) (the output voltage) constant and measure \( V_{in}/I_{in} \).

Unfortunately the \( h \) parameters are not constant.
- e.g. Figs. 11-14 of the 2N3904 spec sheet show the variation of the parameters with \( I_C \).
There are 3 sets of the 4 hybrid parameters.
- One for each type of amp: common emitter, common base, common collector
- In order to differentiate one set of parameters from another the following notation is used:

<table>
<thead>
<tr>
<th>First subscript</th>
<th>Second subscript</th>
</tr>
</thead>
<tbody>
<tr>
<td>i = input impedance</td>
<td>e = common emitter</td>
</tr>
<tr>
<td>o = output admittance</td>
<td>b = common base</td>
</tr>
<tr>
<td>r = reverse voltage ratio</td>
<td>c = common collector</td>
</tr>
<tr>
<td>f = forward current ratio</td>
<td></td>
</tr>
</tbody>
</table>

For a common emitter amplifier we would write:

\[
dV_i = h_{ie} dI_i + h_{re} dV_o \\
dI_o = h_{fe} dI_i + h_{oe} dV_o
\]

Typical values for the \( h \) parameters for a 2N3904 transistor in the common emitter configuration:
\( h_{fe} = 120, \ h_{oc} = 8.7 \times 10^{-6} \, \Omega^{-1}, \ h_{ie} = 3700 \, \Omega, \ h_{re} = 1.3 \times 10^{-4} \) for \( I_C = 1 \, \text{mA} \)

The equivalent circuit for a transistor in the common emitter configuration looks like:

- Circle: voltage source
  - the voltage across this element is always equal to \( h_{re} V_o \) independent of the current through it.
- Triangle: current source
  - the current through this element is always \( h_{fe} I_i \) independent of the voltage across the device.
We can use the model to calculate voltage/current gain and the input/output impedance of a CE amp.

- Equivalent circuit for a CE amp with a voltage source (with resistance $R_s$) and load resistor ($R_{load}$):

  - **Current gain:** $G_I = I_o/I_{in}$
    - Using Kirchhoff’s current law at the output side we have:
      $$h_{fe} I_{in} + V_o h_{oc} = I_o$$
    - Using Kirchhoff’s voltage rule at the output we have:
      $$V_o = -I_o R_{load}$$
      $$h_{fe} I_{in} = h_{oe} I_o R_{load} + I_o$$
      $$G_I = I_o/I_{in} = h_{fe} / (1 + h_{oe} R_{load})$$
    - For typical CE amps, $h_{oe} R_{load} << 1$ and the gain reduces to familiar form:
      $$G_I = h_{fe} = \beta$$

  - **Voltage gain:** $G_V = V_o/V_{in}$
    - This gain can be derived in a similar fashion as the current gain:
      $$G_V = V_o/V_{in} = -h_{fe} R_{load} / (\Delta R_{load} + h_{ie})$$
      with $\Delta = h_{ie} h_{oc} - h_{fe} h_{re} \approx 10^{-2}$
    - This reduces to a familiar form for most cases where $\Delta R_{load} << h_{ie}$
      $$G_V = -h_{fe} R_{load} / h_{ic} = -R_{load} / r_{BE}$$
**Input Impedance:** \( Z_i = \frac{V_{in}}{I_{in}} \)
\[ Z_i = \frac{\Delta R_{load} + h_{ie}}{(1 + h_{oe} R_{load})} \]
- This reduces to a familiar form for most cases where \( \Delta R_{load} \ll h_{ie} \) and \( h_{oe} R_{load} \ll 1 \)
\[ Z_i = h_{ie} = h_{fe} r_{BE} \]

**Output Impedance:** \( Z_o = \frac{V_o}{I_o} \)
\[ Z_o = \frac{R_s + h_{ie}}{(\Delta + h_{oe} R_s)} \]
- \( Z_o \) does not reduce to a simple expression.
- As the denominator is small, \( Z_o \) is as advertised large.

**Feedback and Amplifiers**
- Consider the common emitter amplifier shown.
- This amp differs slightly from the CE amp we saw before:
  - bias resistor \( R_2 \) is connected to collector resistor \( R_1 \) instead of directly to \( V_{cc} \).
- How does this effect \( V_{out} \)?
  - If \( V_{out} \) decreases (moves away from \( V_{cc} \))
    - \( I_2 \) increases
    - \( V_B \) decreases (gets closer to ground)
    - \( V_{out} \) will increase since \( \Delta V_{out} = -\Delta V_B R_1/R_E \)
  - If \( V_{out} \) increases (moves towards \( V_{cc} \))
    - \( I_2 \) decreases
    - \( V_B \) increases (moves away from ground).
    - \( V_{out} \) will decrease since \( \Delta V_{out} = -\Delta V_B R_1/R_E \)

This is an example of NEGATIVE FEEDBACK
Negative Feedback is **good**:  
- Stabilizes amplifier against oscillation  
- Increases the input impedance of the amplifier  
- Decreases the output impedance of the amplifier  
Positive Feedback is **bad**:  
- Causes amplifiers to oscillate

**Feedback Fundamentals:**

Without feedback the output and input are related by:

\[ V_{\text{out}} = AV_{\text{in}} \]

The feedback (box B) returns a portion of the output voltage to the amplifier through the "mixer".

The feedback network on the AM radio is the collector to base resistors \((R_3, R_5)\)

The input to the amplifier is:

\[ V_x = V_{\text{in}} + BV_{\text{out}} \]

The gain with feedback is:

\[ V_{\text{out}} = AV_x = A(V_{\text{in}} + BV_{\text{out}}) \]
\[ G = V_{\text{out}} / V_{\text{in}} = A / (1 - AB) \]

**Oscillation** is a large fluctuation of output signal with no input

\(A\): open loop gain  
\(AB\): loop gain  
\(G\): closed loop gain

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● Positive and negative feedback:

✔ Lets define $A > 0$ (positive)

\[ G = V_{\text{out}} / V_{\text{in}} = A / (1 - AB) \]

✔ Positive feedback, $AB > 0$:

- As $AB \to 1$, $G \to \infty$.
  - circuit is unstable
  - oscillates if $AB = 1$

✔ Negative feedback, $AB < 0$:

- As $A \to \infty$, an amazing thing happens:
  \[ |AB| \to \infty \]
  \[ |G| \to |1/B| \]

- Example: $A = 10^5$ and $B = -0.01$ then $G = 100$.
- The stability of the gain is determined by the feedback loop ($B$) and not the amplifier ($A$).
- Example: $B$ is held fixed at 0.01 and $A$ varies:

<table>
<thead>
<tr>
<th>$A$</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^3$</td>
<td>98.3</td>
</tr>
<tr>
<td>$1 \times 10^4$</td>
<td>99.0</td>
</tr>
<tr>
<td>$2 \times 10^4$</td>
<td>99.6</td>
</tr>
</tbody>
</table>

☞ circuits can be made stable with respect to variations in the transistor characteristics as long as $B$ is stable.

- $B$ can be made from precision components such as resistors.

For large amplifier gain ($A$) the circuit properties are determined by the feedback loop.
Operational Amplifiers (Op Amps)

- Op amps are very high gain \(A = 10^5\) differential amplifiers.
  - Differential amp has two inputs \((V_1, V_2)\) and output \(V_{out} = A(V_1 - V_2)\) where \(A\) is the amplifier gain.

![Op Amp Diagram]

- If an op amp is used without feedback and \(V_1 \neq V_2\)
  - \(V_{out}\) saturates at the power supply voltage (either positive or negative supply).
- Example: Assume the maximum output swing for an op amp is ±15 V.
  - If there is no feedback in the circuit:
    - \(V_{out} = 15\) V if \(V_{\text{non-invert}} > V_{\text{invert}}\)
    - \(V_{out} = -15\) V if \(V_{\text{non-invert}} < V_{\text{invert}}\)

- Op amps are almost always used with negative feedback.
  - The output is connected to the (inverting) input.
- Op amps come in “chip” form. They are made up of complex circuits with 20-100 transistors.

<table>
<thead>
<tr>
<th>Ideal Op Amp</th>
<th>Real Op Amp μA741</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage gain (open loop) (\infty)</td>
<td>(10^5)</td>
</tr>
<tr>
<td>Input impedance (\infty)</td>
<td>2 MΩ</td>
</tr>
<tr>
<td>Output impedance 0</td>
<td>75 Ω</td>
</tr>
<tr>
<td>Slew rate (\infty)</td>
<td>0.5 V/μsec</td>
</tr>
<tr>
<td>Power consumption 0</td>
<td>50 mW</td>
</tr>
<tr>
<td>(V_{out}) with (V_{in} = 0) 0</td>
<td>2 mV (unity gain)</td>
</tr>
<tr>
<td>Price 0$</td>
<td>$0.25</td>
</tr>
</tbody>
</table>

Slew rate is how fast output can change

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When working with op amps using negative feedback two simple rules (almost) always apply:

- **No current goes into the op amp.**
  - This reflects the high input impedance of the op amp.
- **Both input terminals of the op amp have the same voltage.**
  - This has to do with the actual circuitry making up the op amp.

Some examples of op amp circuits with negative feedback:

- **Voltage Follower:**

  ![Voltage Follower Diagram]

  - The feedback network is just a wire connecting the output to the input.
  - By rule #2, the inverting (-) input is also at $V_{in}$.
    - $V_{out} = V_{in}$.
  - What good is this circuit?
    - Mainly as a buffer as it has high input impedance ($M\Omega$) and low output impedance (100 $\Omega$).
Inverting Amplifier:

- By rule #2, point A is at ground.
- By Rule #1, no current is going into the op amp.
- We can redraw the circuit as:

\[
\frac{V_{\text{in}}}{R_1} + \frac{V_{\text{out}}}{R_f} = 0
\]

\[
V_{\text{out}} / V_{\text{in}} = -\frac{R_f}{R_1}
\]

- The closed loop gain is \(R_f/R_1\).
- The minus sign in the gain means that the output has the opposite polarity as the input.
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Non-Inverting Amplifier:

- By rule #2, point A is $V_{in}$.
- By Rule #1, no current is going into the op amp.
- We can redraw the circuit as:

\[
\frac{V_{in}}{R_1} + \frac{(V_{in} - V_{out})}{R_f} = 0
\]

\[
\frac{V_{out}}{V_{in}} = \frac{R_1 + R_f}{R_1}
\]

- The closed loop gain is $(R_1 + R_f) / R_1$.
- The output has the same polarity as the input.
Integrating Amplifier:

Again, using the two rules for op amp circuits we redraw the circuit as:

\[
\frac{V_{\text{in}}}{R_1} + \frac{dQ}{dt} = 0
\]
\[
\frac{V_{\text{in}}}{R_1} + C \frac{dV_{\text{out}}}{dt} = 0
\]
\[
V_{\text{out}} = -\frac{1}{CR_1} \int V_{\text{in}} dt
\]

- The output voltage is related to the integral of the input voltage.
- The negative sign in the gain means that \( V_{\text{in}} \) and \( V_{\text{out}} \) have opposite polarity.
Op Amps and Analog Calculations:
- Op amps were invented before transistors to perform analog calculations.
- Their main function was to solve differential equations in real time.
- Example: Suppose we wanted to solve the following:
  \[
  \frac{d^2x}{dt^2} = g
  \]
  This describes a body under constant acceleration (gravity if \( g = 9.8 \text{ m/s}^2 \)).
- The following circuit gives an output which is the solution to the differential equation:

The input voltage is a constant (\( = g \)).
- For convenience we pick \( RC = 1 \).
- At point A:
  \[
  V_A = -\int V_{in} \, dt = -\int \frac{d^2x}{dt^2} \, dt = -\frac{dx}{dt}
  \]
- The output voltage (\( V_{out} \)) is the integral of \( V_A \):
  \[
  V_{out} = -\int V_A \, dt = \int \frac{dx}{dt} \, dt = x(t)
  \]

If we want non-zero boundary conditions (e.g. \( V(t = 0) = 1 \text{ m/s} \)) we add a DC voltage at point A.