Algebra for Einstein-Lorentz Transformation Derivation

$$K^2(x-vt)^2+y^2+z^2 = c^2(At+Bx)^2$$

or: $x^2(K^2-c^2B^2)+t^2(K^2v^2-A^2c^2)-2xt(K^2v+ABc^2)+y^2+z^2=0$

this must hold at all points in space + time + must be of form $x^2+y^2+z^2=c^2t^2$

$\Rightarrow$ $\underline{\underline{K^2-c^2B^2=1}}$ $\qquad$ $\underline{A^2c^2-K^2v^2=c^2}$ $\qquad$ $\underline{K^2v+ABc^2=0}$

step ①

$$A^2=\frac{c^2+K^2v^2}{c^2} \longrightarrow \text{②}\; K^2v+\left(\frac{c^2+K^2v^2}{c^2}\right)^{1/2}Bc^2=0$$

③

$$K^2-\frac{c^2K^4v^2}{\left(\frac{c^2+K^2v^2}{c^2}\right)c^4}=1 \xleftarrow{\text{④}} B=\frac{-K^2v}{\left(\frac{c^2+K^2v^2}{c^2}\right)^{1/2}c^2}$$

$$K^2(c^2+K^2v^2)-K^4v^2=c^2+K^2v^2$$

$$K^2c^2+\cancel{K^4v^2}-\cancel{K^4v^2}=c^2+K^2v^2$$

$$K^2=\frac{c^2}{c^2-v^2}=\frac{1}{1-v^2/c^2}$$

$$\boxed{K=\sqrt{\frac{1}{1-v^2/c^2}}=\gamma}$$

$$A^2=\frac{c^2+\frac{c^2}{c^2-v^2}v^2}{c^2}=1+\frac{v^2}{c^2-v^2}=\frac{c^2}{c^2-v^2}=K^2$$

$$\boxed{A=K}$$

$$B=-\frac{K^2v}{Ac^2}=-\frac{Kv}{c^2}$$

$$\boxed{B=\frac{-Kv}{c^2}}$$

assumed forms

$$x'=K(x-vt)$$

$$t'=At+Bx=Kt-\frac{Kv}{c^2}x=K\left(t-\frac{vx}{c^2}\right)$$