Physics 133: Midterm I
Spring 2005

Professor Frank De Lucia

3:30 Section
Afternoon Recitation

Name (1 pt): Key

Recitation Instructor (1 pt): Corwin

There are four pages to this midterm (plus an equation sheet). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point.

Be sure to include the proper units in your answers.

1 (25 pts). A 4 kg object on a horizontal frictionless surface is attached to a spring and oscillates with an angular frequency of $\omega = 3$ rad/sec. The amplitude of oscillation is 1 m.

(a) What is the spring constant?

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2 = (4\,kg)\left(3\,\frac{1}{s}\right)^2 = \boxed{36\,\frac{N}{m}}$$

(b) What is the total energy of the system?

$$E_{total} = \frac{1}{2}mV_m^2 = \frac{1}{2}m(\omega X_m)^2 = \frac{1}{2}(m\omega^2)X_m^2 = \frac{1}{2}kX_m^2$$

$$= \frac{1}{2}\left(36\,\frac{N}{m}\right)(1\,m)^2 = \boxed{18\,J}$$

(c) What is the velocity of the mass at $x = 0$ (the equilibrium point)?

$$x(t) = X_m\cos(\omega t + \phi) \Rightarrow v(t) = -\omega X_m \sin(\omega t + \phi)$$

$$\Rightarrow \boxed{V_m = \omega X_m = \left(3\,\frac{1}{s}\right)(1\,m) = 3\,\frac{m}{s}}$$

(d) If the system at t = 0 is at x = 0 and moving towards positive x, write in the form $x = x_m\cos(\omega t + \phi)$ the equation which describes the motion.

$$x(t) = X_m\cos(\omega t + \phi)$$

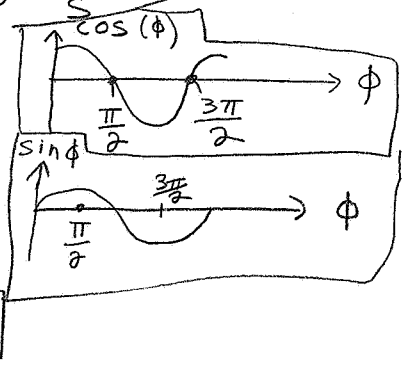
$X_m = 1\,m$

$\omega = 3\,\frac{rad}{s}$ $\quad \phi = ?$

Given $x(0) = 0 \Rightarrow \cos(\phi) = 0$

$\Rightarrow v(0) > 0 \Rightarrow -\omega X_m \sin(\phi) > 0$

$\Rightarrow \sin(\phi) < 0$



$\Rightarrow \phi = \frac{3\pi}{2}$

$$x(t) = (1\,m)\cos\left[\left(3\,\frac{rad}{s}\right)t + \frac{3\pi}{2}\right]$$

Name (1 pt): KEY

2 (25 pts). A traveling wave is described by

$$y(x,t) = (2.0m)\sin\left[(5\pi\ rad/m)x - (30\pi\ rad/s)t\right]$$

(a) What is the period of the wave?

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{30\pi} = \frac{1}{15}\ s$$

(b) What is the velocity of the wave?

$$v = \frac{\omega}{k} = \frac{30\pi}{5\pi} = 6\frac{m}{s}$$

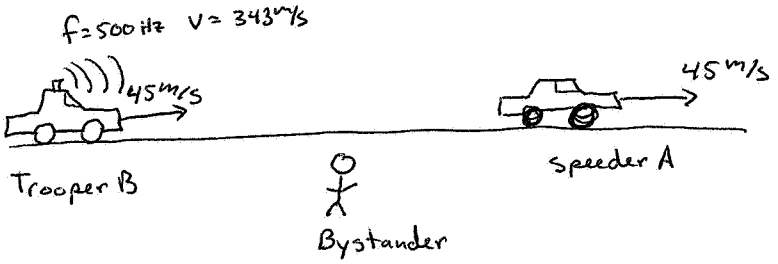
(c) What wave would have to be added to this wave to make a pure standing wave?

$$y_2(x,t) = 2.0m \sin\left[5\pi x + 30\pi t\right]$$

(traveling in opposite direction)

Name (1 pt): Key

3 (20 pts). Trooper *B* is chasing speeder *A* along a straight stretch of road. Both are moving at a speed of 45 m/s. Trooper *B*, failing to catch up, sounds his siren again. Take the speed of sound in air to be 343 m/s and the frequency of the source to be 500 Hz.



(a) What frequency is heard by a bystander standing along side the road? (10 pts)

If bystander is between B and A, source (B) is moving towards bystander ∴ shift up

$$f' = f\,\frac{v \pm v_D}{v \pm v_S} = 500\,\text{Hz}\,\frac{343\ \text{m/s}}{(343-45)\ \text{m/s}} = \underline{575.5\ \text{Hz}}$$

If bystander is behind trooper B, source is moving away from bystander ∴ shift down

$$f' = f\,\frac{v \pm v_D}{v \pm v_S} = 500\,\text{Hz}\,\frac{343\ \text{m/s}}{(343+45)\ \text{m/s}} = \underline{442\ \text{Hz}}$$

(b) What frequency is heard by the speeder? (10 pts)

The trooper and speeder are going the same direction and velocity, ∴ there will be no doppler shift. Speeder A hears 500 Hz

Looking at the math;

$$f' = f\,\frac{v \pm v_D}{v \pm v_S} = 500\,\text{Hz}\,\frac{(343-45)\ \text{m/s}}{(343-45)\ \text{m/s}} = 500\,\text{Hz}$$

(343−45) m/s → speeder A moving "away" from source ∴ shift down

(343−45) m/s → source moving "towards" speeder ∴ shift up

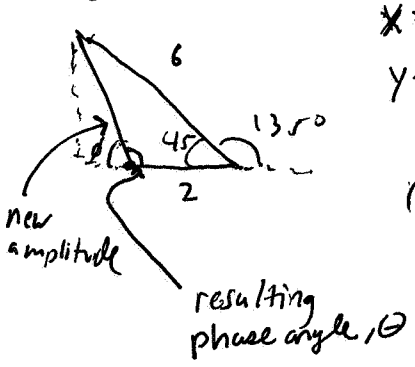
Name (1 pt): KEY

4 (25pts). The amplitudes of two sinusoidal waves traveling in the same direction along a string are 2.0 and 6.0 mm; The phase angle for the first wave is 0 and for the second 0.75π (135^0) . The waves have the same wavelength.

(a) Is the resulting wave a standing or traveling wave?

Traveling

(b)What is the amplitude of the resulting wave?



$X = 2\cos 0 + 6\cos 135 = 2 + -4.24 = -2.24$

$Y = 2\sin 0 + 6\sin 135 = 0 + 4.24 = 4.24$

Resulting amplitude $= \sqrt{(-2.24)^2 + (4.24)^2}$

$= 4.80$ mm

(c) What is the phase angle of the resulting wave?

$$\tan\phi = \frac{4.24}{-2.24}$$

$$\phi = \tan^{-1}\left(\frac{4.24}{-2.24}\right) = 62^\circ$$

so $\theta = 180^\circ - 62^\circ = 118^\circ$