Spring | Physics 133: Midterm I
Professor Frank De Lucia | 2:30 Section

Name (1 pt): KEY

Recitation Instructor (1 pt):______________________________

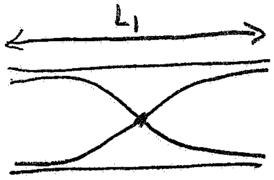
There are four pages to this midterm (plus an equation sheet). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point.

Be sure to include the proper units in your answers.

1 (25 pts): There are two organ pipes of length $L_1 = 2$ m and $L_2 = 3$ m. The pipe of length L_1 is open on both ends, and the pipe of length L_2 has one end open and one end closed. The speed of sound is 343 m/s.

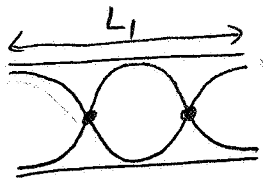
(a) What are the two lowest resonant frequencies for the pipe of length L_1? Draw the pipe and sketch the standing waves.

First Harmonic:
(Fundamental)



$$\lambda_1 = 2L_1, \quad f_1 = \frac{V_s}{\lambda_1} = \frac{V_s}{2L_1} = \frac{343\,\text{m/s}}{2(2\,\text{m})} = \boxed{85.8\,\text{Hz}}$$

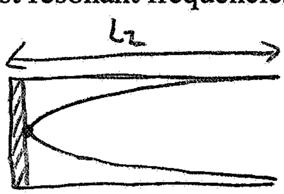
Second Harmonic:
(First overtone)



$$\lambda_2 = L_1, \quad f_2 = \frac{V_s}{\lambda_2} = \frac{V_s}{L_1} = \frac{343\,\text{m/s}}{2\,\text{m}} = \boxed{172\,\text{Hz}}$$

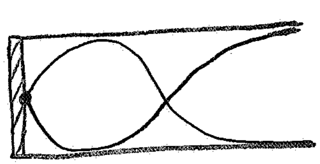
(b) What are the two lowest resonant frequencies for the pipe of length L_2? Draw the pipe and sketch the standing waves.

First Harmonic:
(Fundamental)



$$\lambda_1 = 4L_2, \quad f_1 = \frac{V_s}{\lambda_1} = \frac{V_s}{4L_2} = \frac{343\,\text{m/s}}{4(3\,\text{m})} = \boxed{28.6\,\text{Hz}}$$

Third Harmonic:
(First overtone)



$$\lambda_3 = \frac{4L_2}{3}, \quad f_3 = \frac{V_s}{\lambda_3} = \frac{3V_s}{4L_2} = \frac{3(343\,\text{m/s})}{4(3\,\text{m})} = \boxed{85.8\,\text{Hz}}$$

Name (1 pt) ____________________________

2 (25 pts)

a) Write an equation describing a sinusoidal transverse wave traveling on a string in the negative x direction with a wavelength of 0.5m, a frequency of 200 Hz, and an amplitude of 3.0 cm. You do not need to embed the units in the equation you write.

$y(x,t) = Y_m \sin(kx - \omega t)$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\frac{1}{2}\,m} = 4\pi\ m^{-1}$

$\omega = 2\pi f = (2\pi)(200\ Hz) = 400\pi\ rad/s$

$= (3.0cm)\sin(4\pi x - 400\pi t)$

(b) What is the maximum transverse speed of a point on the cord?

$V_y = \frac{dy}{dt} = (3.0cm)\underbrace{(-\omega)\cos(kx - \omega t)}_{\text{max value} = \omega}$

$V_y = (3.0cm)(400\pi\ rad/s) = 1200\pi\ cm/s$

(c) What is the speed of the wave?

$V = f\lambda = (200\ Hz)(0.5m) = 100\ m/s$

Name (1 pt) 2:30 Lecture P.3

3 (20 pts). An oscillating block-spring system has a mechanical energy of 1.00 J, an amplitude of 10.0 cm, and a maximum speed of 1.20 m/s.

$u_m = 1.2\ m/s$ $E = 1\ J$ $y_m = 0.1\ m$

(a) Find the spring constant.

$$E = \frac{1}{2} k y_m^2$$

$$k = \frac{2E}{y_m^2} = \frac{2 \times 1}{(0.1)^2} = 200\ N/m$$

(b) Find the mass of the block.

$$\omega = \sqrt{\frac{k}{m}}, \quad \omega = \frac{u_m}{y_m}$$

$$\therefore\ m = k\left(\frac{y_m}{u_m}\right)^2 = 200 \times \left(\frac{0.1}{1.2}\right)^2 = \frac{25}{18}\ kg$$

$$\approx 1.39\ kg$$

(c) Find the frequency of oscillation.

$$f = \frac{\omega}{2\pi} = \frac{u_m}{y_m} \cdot \frac{1}{2\pi} = \frac{1.2}{0.1} \times \frac{1}{2\pi} = \frac{6}{\pi}\ Hz$$

$$\approx 1.91\ Hz$$

Name (1 pt) ____________________

4 (25 pts). A standing wave is described by

$$y(x,t) = (2.0\,cm)\sin\left[(4\pi\,rad/m)x\right]\cos\left[(10\pi\,rad/s)t\right]$$

(a) What is the wavelength of one of the traveling waves that contributes to this standing wave?

The traveling waves must have the same wavelength as the standing wave itself, given by

$$\lambda = \frac{2\pi}{k}$$

We can read off $k = 4\pi\,rad/m$, so

$$\lambda = \frac{2\pi}{4\pi\,m^{-1}} = \boxed{\frac{1}{2}\,m}$$

(b) What is the maximum transverse velocity of a point on the string?

With $y(x,t) = y_m \sin(kx)\cos(\omega t)$, the transverse velocity is $u(x,t) = \frac{\partial}{\partial t} y_m \sin(kx)\cos(\omega t)$

$$= -\omega y_m \sin(kx)\sin(\omega t)$$

This has its maximum magnitude when $\sin(kx) = \pm 1$ and $\sin(\omega t) = \pm 1$:

$$u_{max} = \left|-\omega y_m (\pm 1)(\pm 1)\right| = \omega y_m = (10\pi\,\tfrac{rad}{s})(2.0\,cm)$$

$$= \boxed{20\pi\,cm/s}, \text{ or } 0.628\,m/s$$

Here, y_m is used as the amplitude of the standing wave. Also OK is to use $y(x,t) = 2y_m \sin(kx)\cos(\omega t)$ $\Rightarrow u_m = 2\omega y_m$ where $y_m = 1.0\,cm$ is the amplitude of one of the traveling waves.

(c) What is the minimum transverse velocity of a point on the string?

$u(x,t)$ will have its minimum magnitude when either $\sin(kx) = 0$ (at a node) or $\sin(\omega t) = 0$ (when the string is at maximum displacement), so that $\boxed{u_{min} = 0}$ is the minimum speed.

Also OK is to read the question as askin for the minimum of the velocity as a signed value, in which case $\boxed{u_{min} = -u_{max}}$.

(remember that a radian is a dimensionless unit)