11:30 Section
Spring 2004
Professor Frank De Lucia

Physics 133: Midterm II

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Total ____________

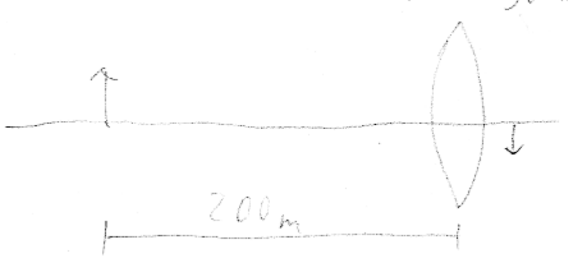
Name (1 pt):______________________________

Recitation Instructor (1 pt):______________________________

There are four pages to this midterm (plus an equation sheet). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point.

Be sure to include the proper units in your answers.

Problem I.1 (10 pts): A camera with a lens whose focal length is +50 mm takes a picture of a tree that is 2 m high and 200 m from the camera. What is the size (without a sign) of the image of the tree on the film in the camera? You may assume that because 200 m >> 50 mm, that the distance between the film and the lens is 50 mm.

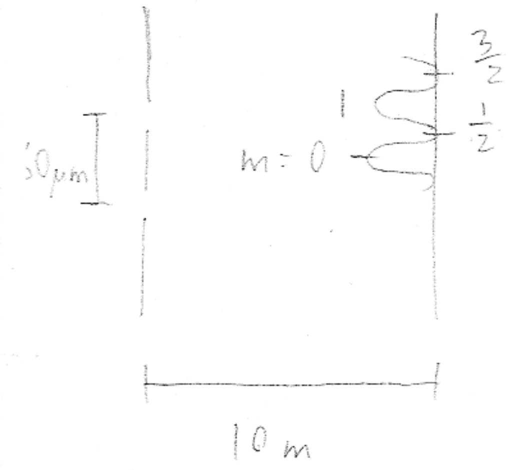


$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} \Rightarrow i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = 50\,\text{mm}$$

$$m = \frac{-i}{p} = \frac{-50\,\text{mm}}{200\,\text{m}} = -0.00025$$

$$\text{height} = |m| \cdot 2\,\text{m}$$
$$= 0.00025 \cdot 2\,\text{m}$$
$$= 5 \times 10^{-4}\,\text{m} = 0.5\,\text{mm}$$

Problem I.2 (10 pts): A double-slit experiment has slits separated by 60 μm, and is illuminated with light of λ = 600 nm. On a wall located 10 m from the slits, how far apart are the first and second minima?



$$d \sin\theta = \frac{1}{2}\lambda \qquad \sin\theta \approx \tan\theta = \frac{x}{L}$$

$$d\frac{x_1}{L} = \frac{1}{2}\lambda \Rightarrow x_1 = \frac{\lambda L}{2d}$$

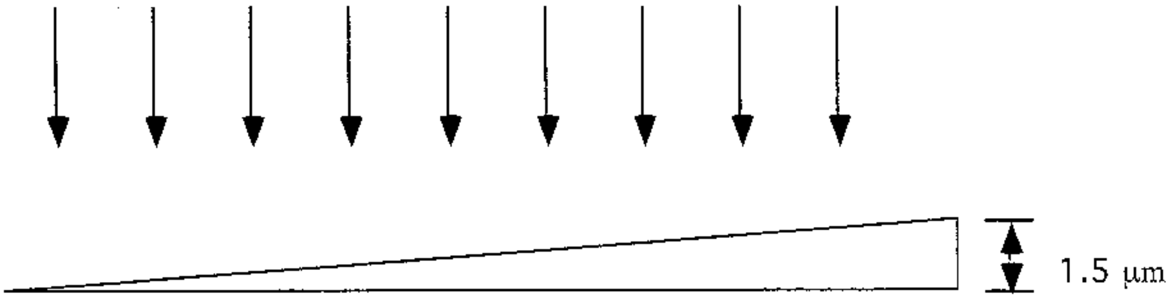
$$d\sin\theta = \frac{3}{2}\lambda \Rightarrow d\frac{x_2}{L} = \frac{3}{2}\lambda \Rightarrow x_2 = \frac{3\lambda L}{2d}$$

$$x_2 - x_1 = \frac{3\lambda L}{2d} - \frac{\lambda L}{2d} = \frac{\lambda L}{d}$$

$$= \frac{(600 \times 10^{-9}\,\text{m})(10\,\text{m})}{(60 \times 10^{-6}\,\text{m})} = 0.1\,\text{m}$$

Name (1 pt):____________________________

Problem II.1 (25 pts). A broad beam of light of wavelength 450 nm is sent directly downward through the top of a thin wedge of glass of n = 1.5 as shown in the figure. The wedge is surrounded by air. The wedge has an height of 1.5 μm and an unspecified width. How many bright fringes will be seen by an observer looking at the reflected light from above?



relative phase change on reflection

so for bright fringe must have:

path length difference $\underbrace{2t}_{\text{thickness of glass}} = (\underbrace{m}_{\text{integer}} + \tfrac{1}{2})$ wavelengths in glass $\left(\frac{\lambda}{n}\right)$

max thickness = 1.5 μm

$$\frac{2t_{max}}{\lambda/n} = \frac{2(1.5 \times 10^{-6})}{\frac{4.5 \times 10^{-7}}{1.5}} = 10$$

ie $2 \times 1.5\ \mu m = 10$ wavelengths in wedge

(this will be dark)

bright fringes appear for $\frac{1}{2}, \frac{3}{2}, \ldots 9\frac{1}{2}$ wavelengths

$$\underbrace{m = 0, 1, \ldots, 9}_{\text{ten fringes}}$$

Name $+1$ ↙ no name spot $+1$ Auto

Problem II.2 (25 pts). A single slit with width 4.5 μm is illuminated with light of wavelength λ = 450 nm.

6 (a) What is the angular location of the first minimum in the diffraction pattern?

$a\sin\theta = m\lambda$ $\quad m=1$ (1st minimum)

$\sin\theta = \frac{\lambda}{a}$ $\quad \Rightarrow \sin\theta = 1/10$

ie $\sin\theta \approx \theta$.

$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right) \Rightarrow \theta = .1002$ rad $= 5.73°$

6 (b) What is the angular location of the second minimum in the diffraction pattern?

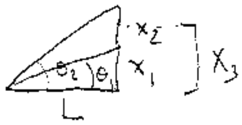
$m=2$ $\quad a\sin\theta = m\lambda$

$\sin\theta = \frac{2\lambda}{a}$ $\quad \Rightarrow \sin\theta = 2/10$

$\theta = \sin^{-1}\left(\frac{2\lambda}{a}\right) \Rightarrow \theta = .2014$ rad $= 11.54°$

13 (c) What is the ratio of the light intensity at a position half way between the first and second minima to the intensity of the central maximum?

Since small angle holds $\tan\theta \approx \sin\theta \approx \theta$



$X_3 = \frac{X_2 - X_1}{2} + X_1 = \frac{X_2 + X_1}{2} = \frac{\theta_2 \cdot L + \theta_1 \cdot L}{2} = L\,\frac{\theta_2 + \theta_1}{2}$

note $\frac{X_1}{L} \approx \theta_1$ $\quad \theta_3 = \frac{X_3}{L}$ $\quad$ so $\theta_3 = \frac{\theta_2 + \theta_1}{2}$ (as expected)

Small angle approx $\rightarrow .15$

$I_m \equiv$ Intensity of central maximum.

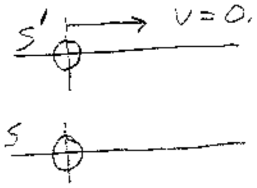
$\frac{I_3}{I_m} = \frac{I_m}{I_m}\left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin\alpha}{\alpha}\right)^2$ $\quad \alpha = \frac{\pi a}{\lambda}\sin\theta_3$ $\quad \theta_3 = .1508 = 8.64°$ (.1, .2)

$= 10\pi \cdot \sin(.1508) = 1.502\pi$ (rad) ≈ 4.72 rad

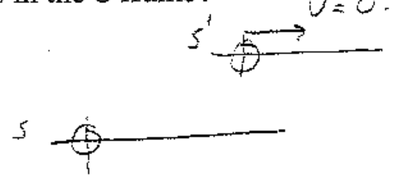
$= \left(\frac{-.99997}{4.72}\right)^2 = .0449$ //

Problem II.3 (25 pts). A coordinate system S' moves to the right relative to S at v = 0.6 c. As a clock at x' = 0 in the S' frame passes x = 0 of the S frame, it is set to zero, as is a clock at x = 0 in the S frame. Additionally, at this event (i.e. the passing of the clock at the origin of S' past the origin of the S frame) all of the clocks within S are synchronized. After 500 seconds go by on the clock at the origin of S'

(a) Where is this clock according to observers in the S frame?



$x = x' = 0$
$t = t' = 0$



$t_2' = 500\,s$
$x_2' = 0$

$x_2 = \gamma(x_2' + v t_2')$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1.25$$

$$x_2 = 1.25\,(0 + 0.6\,(3\times10^8\,m/s)(500\,s))$$
$$= 1.25\,(1.8)(5)\,(10^{10})$$
$$\boxed{x_2 = 1.125\times10^{11}\,m}$$

(b) According to observers in S, how much time has elapsed?

$$t_2 = \gamma\left(t_2' + \frac{v x_2'}{c^2}\right) = 1.25\,(500\,s) = 625\,s$$

(c) According to observes in S, what is the speed of the clock at the origin of S'?

In S, S' (and the clock) move at 0.6 c

or

$$V = \frac{x_2}{t_2} = \frac{1.125\times10^{11}\,m}{625\,s} = 1.8\times10^8\,m/s = 0.6\,c$$