

Physics 133  
Final Exam  
1:30 - 3:18 PM, Tuesday, June 6

Spring 2000

2:30 Section

Name (1 pt): \_\_\_\_\_

Recitation Instructor (1 pt): \_\_\_\_\_

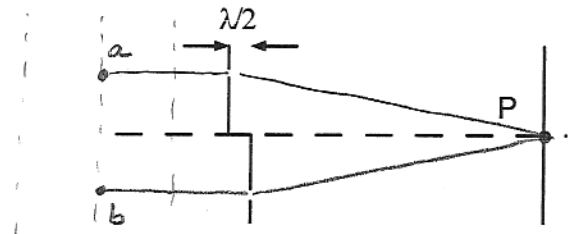
This test consists of a total of 9 pages, 7 of problems, an optional question about course format, and an equation sheet.

Section I - short problems/questions

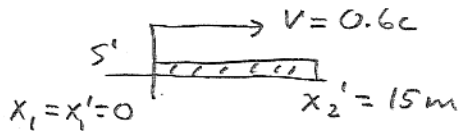
I.1 (10 pts) A double slit experiment is modified as shown in the figure so that one of the slits is  $\lambda/2$  farther to the right than the other. At "P", the central location on the screen, is there a maximum or a minimum?

Why? A maximum

The distance ( $\pm$  phase) from "a" to "P" and "b" to "P" are the same  $\Rightarrow$  in phase  $\pm$  maximum at "P"



I.2 (10 pts): A stick of length  $l = 15$  m in the  $S'$  frame is moving at  $v = 0.6c$  relative to observers in the  $S$  frame. How long do observers in the  $S$  frame measure the stick to be?



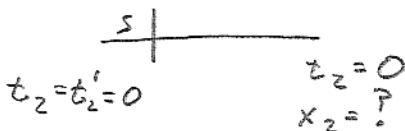
method: make measurement simultaneously in  $S$  at  $t_1 = t_2 = 0$

$$x_2' = \gamma(x_2 - vt_2)$$

$$15\text{m} = 1.25(l_S - 0.6c \cdot 0)$$

$$\boxed{l_S = \frac{15\text{m}}{1.25} = 12\text{m}}$$

$$\gamma = \frac{1}{\sqrt{1 - 0.36}} = 1.25$$



I.3 (10 pts): As measured in parsecs, how far away is a star whose parallax is 0.1 arc sec?

$$D(\text{pc}) = \frac{1}{\alpha(")} = \frac{1}{0.1"} = 10\text{pc}$$

Name (1 pt): \_\_\_\_\_

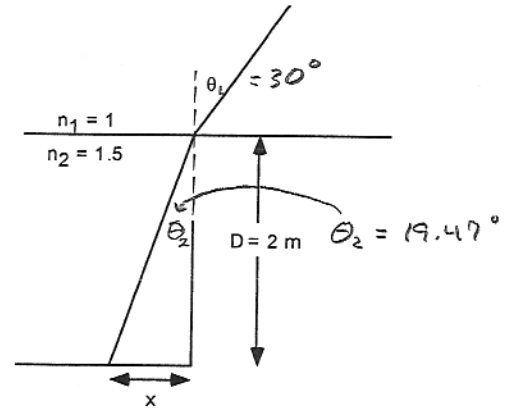
I.4 (10 pts) A ray of light is incident on a pool of depth 2 m at an angle of  $\theta = 30^\circ$  from the normal. The liquid in the pool has an index of refraction of  $n_2 = 1.5$ . As shown in the figure the ray enters the pool at its edge and hits the bottom a distance  $x$  from the edge of the pool. What is  $x$ ?

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$
$$1 \cdot 0.5 = 1.5 \sin \theta_2$$
$$\sin \theta_2 = \frac{1}{3} \Rightarrow \theta_2 = 19.47^\circ$$

$$\tan \theta_2 = \frac{x}{2\text{m}}$$

$$0.3536 = \frac{x}{2\text{m}}$$

$$x = 0.707\text{m}$$



In the following two multiple choice questions, circle the best answer.

I.5 (10 pts): Most of the deuterium in the universe was created

- (a) during the first few minutes after the big bang.
- (b) in the era around the time of the launching of the microwave background radiation.
- (c) in the interiors of stars like the sun.
- (d) in supernova collapses

I.6 (10 pts): Which of the following does NOT significantly effect or predict the future of the universe?

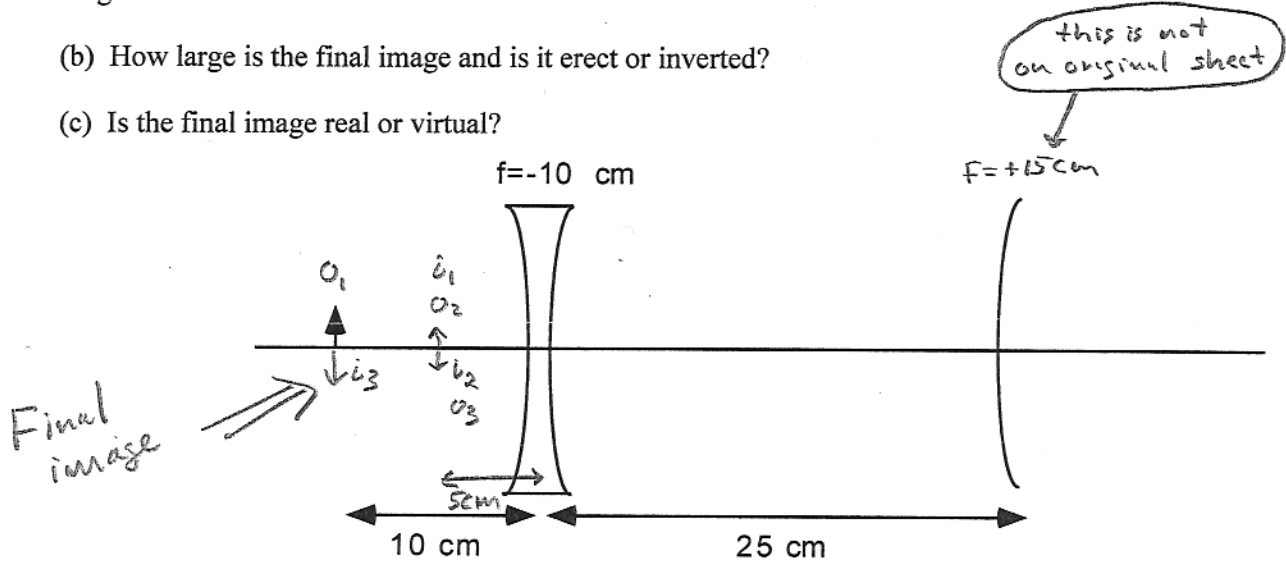
- (a) The mass density of the universe
- (b) the ratio of iron to magnesium in the universe
- (c) The size of the Hubble constant
- (d) the abundance of dark matter in the universe

Name (1 pt): \_\_\_\_\_

Part II.

II.1 (30 pts) The optical system below consists of a lens on the left and a mirror on the right. The object is erect and has a height of 2 cm. After the light has passed the lens for the second time and exited the system

- (a) What is the location of the final image? Unambiguously draw and label this location on the diagram.
- (b) How large is the final image and is it erect or inverted?
- (c) Is the final image real or virtual?



#1 lens 1st time:

$$\frac{1}{10\text{cm}} + \frac{1}{i} = \frac{1}{-10\text{cm}} \Rightarrow i = -5\text{cm} \Rightarrow m_1 = -\frac{i}{p} = \frac{1}{2}$$

#2 mirror:

$$\frac{1}{30\text{cm}} + \frac{1}{i} = \frac{1}{+15\text{cm}}; \frac{1}{i} = \frac{1}{15\text{cm}} - \frac{1}{30\text{cm}} = \frac{1}{30\text{cm}} \Rightarrow i = +30\text{cm} \Rightarrow m_2 = \frac{-30}{30} = -1$$

#3 lens again:

$$\frac{1}{-5\text{cm}} + \frac{1}{i} = \frac{1}{-10\text{cm}}; \frac{1}{i} = \frac{1}{5\text{cm}} - \frac{1}{10\text{cm}} \Rightarrow i = 10\text{cm} \Rightarrow m_3 = -\left(\frac{10}{-5}\right) = +2$$

#3

$$b) m = m_1 m_2 m_3$$

$$= \left(\frac{1}{2}\right)(-1)(+2) = -1$$

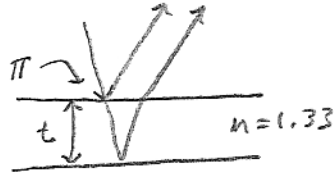
$$\text{Final size} = (2\text{cm})(-1) = -2\text{cm}$$

[inverted]

c) real

Name (1 pt): \_\_\_\_\_

II.2 (25 pts): The reflection of perpendicularly incident white light by a soap film in air has an interference maximum at 600 nm and a minimum at 450 nm, and there are no minimum or maxima between 450 nm and 600 nm. If  $n = 1.33$  for the film, what is the film thickness?



For  $\lambda = 600 \text{ nm}$  maximum

For  $\lambda = 450 \text{ nm}$  minimum

For maximum

$$t = (n + \frac{1}{2}) \frac{\lambda_n}{2}, \quad n = 0, 1, 2, \dots \quad \frac{1}{4}, \frac{3}{4}$$

$$\lambda_n = \frac{600}{1.33} = 450 \text{ nm}$$

$$\lambda_n = \frac{450}{1.33} = 338 \text{ nm}$$

For minimum

$$t = n \frac{\lambda_n}{2}, \quad n = 0, 1, 2, \dots \quad 0, \frac{1}{2}, 1$$

equate

$$(n + \frac{1}{2}) \frac{450 \text{ nm}}{2} = (n + 1) \frac{338 \text{ nm}}{2}$$

while the "n" must be adjacent, they do not need to be the same

$$n \cdot 450 \text{ nm} + 225 \text{ nm} = n \cdot 338 \text{ nm} + 338 \text{ nm}$$

$$n \cdot 112 \text{ nm} = 113 \text{ nm} \Rightarrow n = 1$$

From max:  $t = \frac{3}{2} \frac{450 \text{ nm}}{2} = 337.5 \text{ nm} \checkmark$

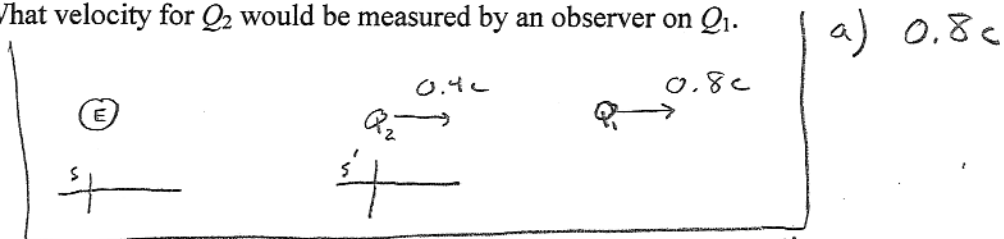
From min:  $t = n + 1 \frac{338 \text{ nm}}{2} = 338 \text{ nm} \checkmark$

Name (1 pt): \_\_\_\_\_

II.3 (25 pts): It is concluded from measurements of the light emitted by quasar  $Q_1$  that it is moving away from us at a speed of  $0.800c$ . Quasar  $Q_2$ , which lies in the same direction in space but is closer to us, is moving away from us at a speed  $0.400c$ .

(a) At what velocity would an observer on  $Q_1$  measure us to be receding?

(b) What velocity for  $Q_2$  would be measured by an observer on  $Q_1$ .



b) the velocity of  $Q_1$  as observed  $S$ :  $x = 0.8c t$

relate to  $S'$ :

$$x = \gamma(x' + vt')$$

$$t = \gamma(t' + \frac{vx'}{c^2})$$

substitute:

$$\cancel{\gamma}(x' + vt') = u \cancel{\gamma}(t' + \frac{vx'}{c^2})$$

$$= ut' + \frac{uvx'}{c^2}$$

$$t'(v-u) = -x'(\frac{uv}{c^2} - 1)$$

$$u' = \frac{x'}{t'} = \frac{u-v}{1 - \frac{uv}{c^2}} = \frac{0.8c - 0.4c}{1 - \frac{(0.8c)(0.4c)}{c^2}}$$

$$= \frac{0.4c}{1 - .32} = \frac{0.4c}{.68} = .588c$$

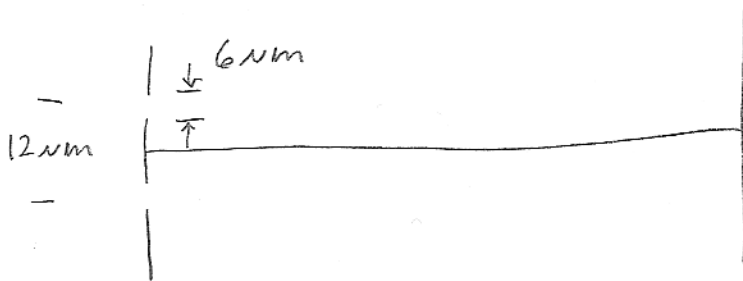
but this is  $Q_1$  as observed by  $Q_2(S')$

$Q_2$  as observed by  $Q_1$  is also receding at  $0.588c$ , but as viewed by  $Q_1$ , to the left

Name (1 pt): \_\_\_\_\_

II.4 (30 pts): A double slit with slit separation  $12 \mu\text{m}$  and slit width  $6 \mu\text{m}$  is illuminated with light of wavelength  $\lambda = 600 \text{ nm}$ .

- What is the angular location of the first side maximum in the interference pattern?
- What is the angular location of the third side maximum in the interference pattern?
- What is the ratio of the light intensities for these two maxima?



For interference pattern:  $d \sin \theta = m \lambda$  - max

$$a) m=1 \Rightarrow \sin \theta = \frac{\lambda}{d} = \frac{.6 \mu\text{m}}{12 \mu\text{m}} = 0.05 \Rightarrow \theta = 2.866^\circ$$

$$b) m=3 \Rightarrow \sin \theta = \frac{3\lambda}{d} = \frac{1.8 \mu\text{m}}{12 \mu\text{m}} = 0.15 \Rightarrow \theta = 8.629^\circ$$

c) This is determined by the diffraction pattern

$$I = I_m \left( \frac{\sin \alpha}{\alpha} \right)^2 \quad \alpha = \frac{\pi a}{\lambda} \sin \theta$$

$$\frac{\pi a}{\lambda} = \frac{\pi 6 \mu\text{m}}{.6 \mu\text{m}} = 10\pi$$

$$I_1 = I_m \left( \frac{\sin \left[ \frac{10\pi \cdot 0.05}{2} \right]}{10\pi \cdot 0.05} \right)^2$$

$$\sin \frac{\pi}{2} = +1$$

$$I_2 = I_m \left( \frac{\sin \left[ \frac{10\pi \cdot 0.15}{2} \right]}{10\pi \cdot 0.15} \right)^2$$

$\underbrace{10\pi \cdot 0.15}_{3\pi/2}$

$$\sin \frac{3\pi}{2} = -1$$

$$\frac{I_1}{I_2} = \left( \frac{3\pi/2}{\pi/2} \right)^2 = 9$$

Name (1 pt): \_\_\_\_\_

II.5 (22 pts): What is the speed of a particle

(a) Whose kinetic energy is equal to three times its rest energy?

(b) Whose total energy is equal to three times its rest energy?  
relate to these questions?

$$a) K = mc^2(\gamma - 1), K = 3mc^2$$

$$3 = 1(\gamma - 1)$$

$$4 = \gamma = \frac{1}{(1 - \frac{v^2}{c^2})^{1/2}} \Rightarrow 16(1 - \frac{v^2}{c^2}) = 1$$

$$\frac{15}{16} = \frac{v^2}{c^2} \quad \boxed{v = \sqrt{\frac{15}{16}} c}$$

$$b) E = \gamma mc^2 \quad E = 3mc^2$$

$$3 = \gamma = \frac{1}{(1 - \frac{v^2}{c^2})^{1/2}} \Rightarrow 9(1 - \frac{v^2}{c^2}) = 1$$

$$\frac{8}{9} = \frac{v^2}{c^2} \quad \boxed{v = \sqrt{\frac{8}{9}} c}$$