Physics 131: FINAL
1:30 - 3:18 pm, Wednesday, December 9, 1998

Fall 1998 | Professor Frank De Lucia | 2:30 Section

Name (1 pt): De Lucia

Recitation Instructor (1 pt):______________________

There are **7** pages to this exam (plus this page). It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point.

Be sure to include the proper units in your answers.

$$a = \frac{v^2}{r} \qquad F_g = \frac{GMm}{r^2} \qquad f_{s,\max} = \mu_s N$$

$$P = \frac{dW}{dt} \qquad U_g = mgh \qquad f_k = \mu_k N$$

$$U_g = -\frac{GMm}{r} \qquad F_s = -kx \qquad U_s = \frac{1}{2}kx^2$$

$g = 9.8\ \text{m/s}^2 = 32\ \text{ft/s}^2$

$G = 6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$

$I = \Sigma m_i r_i^2$

EQUATION NUMBER	LINEAR FORMULA	MISSING VARIABLE		ANGULAR FORMULA	EQUATION NUMBER
(2-11)	$v = v_0 + at$	x	θ	$\omega = \omega_0 + \alpha t$	(11-9)
(2-15)	$x = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	(11-10)
(2-16)	$v^2 = v_0^2 + 2ax$	t	t	$\omega^2 = \omega_0^2 + 2\alpha\theta$	(11-11)
(2-17)	$x = \frac{1}{2}(v_0 + v)t$	a	α	$\theta = \frac{1}{2}(\omega_0 + \omega)t$	(11-12)
(2-18)	$x = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta = \omega t - \frac{1}{2}\alpha t^2$	(11-13)

PURE TRANSLATION (FIXED DIRECTION)		PURE ROTATION (FIXED AXIS)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F = ma$	Newton's second law	$\tau = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power	$P = Fv$	Power	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

TRANSLATIONAL		ROTATIONAL	
Force	$\mathbf{F}$	Torque	$\boldsymbol{\tau}\ (= \mathbf{r} \times \mathbf{F})$
Linear momentum	$\mathbf{p}$	Angular momentum	$\boldsymbol{\ell}\ (= \mathbf{r} \times \mathbf{p})$
Linear momentum[b]	$\mathbf{P}\ (= \Sigma \mathbf{p}_i)$	Angular momentum[b]	$\mathbf{L}\ (= \Sigma \boldsymbol{\ell}_i)$
Linear momentum[b]	$\mathbf{P} = M\mathbf{v}_{cm}$	Angular momentum[c]	$L = I\omega$
Newton's second law[b]	$\Sigma \mathbf{F}_{ext} = \frac{d\mathbf{P}}{dt}$	Newton's second law[b]	$\Sigma \boldsymbol{\tau}_{ext} = \frac{d\mathbf{L}}{dt}$
Conservation law[d]	$\mathbf{P}$ = a constant	Conservation law[d]	$\mathbf{L}$ = a constant

[a] See also Table 11-3. [b] For systems of particles, including rigid bodies.
[c] For a rigid body about a fixed axis, with L being the component along that axis.
[d] For an isolated system.

Name (1 pt) ____________________________

Section I - short problems (10 pts each)

I-1 An outfielder throws a baseball (mass = 0.25 kg) with an initial speed of 30 m/s. Just before an infielder catches the ball at the same height, its speed is 25 m/s. How much of the ball's mechanical energy is dissipated by the air drag acting on the ball during its flight?

Because the infielder catches the ball at the same height, there is no change in potential energy $\Rightarrow$

$$\frac{1}{2} m v_i^2 + W = \frac{1}{2} m v_f^2$$

$$\frac{1}{2}(0.25\,kg)(30\,m/s)^2 + W = \frac{1}{2}(0.25\,kg)(25\,m/s)^2$$

$$112.5\,J + W = 78.125\,J$$

I-2 An instructor stands on a platform that is rotating without friction with an angular velocity of 2 rev/s. Because his arms are outstretched and he holds a weight in each hand the rotational inertia of the system is 10 kg m^2. By moving the weights he reduces the rotational inertia of the system to 5 kg m^2. What is the new angular velocity?

$$L_i = I_i \omega_i = 10\,kg\,m^2 \cdot 2\,rev/s$$

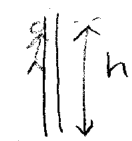
$$L_f = I_f \omega_f = 5\,kg\,m^2 \cdot \omega_f$$

$$L_i = L_f \Rightarrow \omega_f = 4\,rev/s$$

note: although the usual units of angular velocity are rad/s, as long as you are consistent you can use any units

I-3 A 250 kg bear slides, from rest, 12 m down a pine tree, attaining a speed of 6 m/s just before hitting the ground. What is the average frictional force that acts on the bear?

$$E_i = \cancel{\frac{1}{2} m v_i^2}^{\,0} + mgh = (250\,kg)(9.8\,m/s^2)(12\,m) = 29400\,J$$



$$E_f = \frac{1}{2} m v_f^2 + 0 = \frac{1}{2}(250\,kg)(6\,m/s)^2 = 4500\,J$$

$$W = -f_k h$$

$$W = \vec{F} \cdot \vec{d} = -f_k h$$

$$E_i + W = E_f \Rightarrow 29400\,J - f_k(12\,m) = 4500\,J$$

$$\boxed{f_k = 2075\,N}$$

Name (1 pt) ____________________________

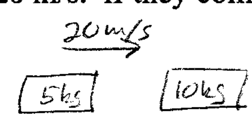
I-4 5 kg block is accelerated by a compressed spring whose spring constant is 600 N/m. After leaving the spring at the spring's relaxed length, the block travels over a horizontal surface, with a coefficient of kinetic friction of 0.25, for a distance of 8 m before stopping. The surface over which the block moves while in contact with the spring is frictionless. Through what distance was the spring compressed before the block began to move?

$E_i = \frac{1}{2} k x^2 \qquad E_f = 0 \qquad W = -(0.25)(5\,kg)(9.8\,m/s^2)\cdot 8\,m = -98\,J$

$E_i + W = 0$

$\frac{1}{2} k x^2 = 98\,J \qquad x = \sqrt{\frac{2 \cdot 98\,J}{600\,N/m}} = .57\,m$

I-5 Two masses rest on a frictionless surface. One is stationary and has a mass of 10 kg and the second has a mass of 5 kg and an initial velocity toward the first of 20 m/s. If they collide and stick together. What is their speed after the collision?

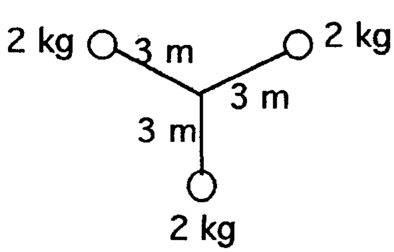


$P_i = (5\,kg)(20\,m/s) = 100\,kg\cdot m/s$

$P_f = (m_1 + m_2) V_f = 15\,kg\; V_f$

$P_i = P_f \Rightarrow 6.67\,m/s$

I-6 Three masses are arranges as shown in the diagram. What is the rotational inertia of this system about its center?



$I = \sum m_i r_i^2 = 6\,kg \cdot (3\,m)^2$

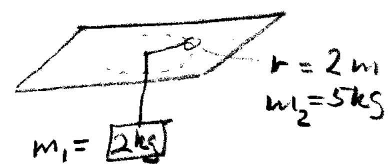
$= 54\,kg\cdot m^2$

Name (1pt) ______________________________

I-7 A uniform beam of length 12 meters and mass 40 kg rests horizontally on scales placed under each of its ends. If a 100 kg mass is placed 3 meters from the left end, how much does each scale read? THIS IS A PROBLEM FROM CPT 12

I-8 A mass of 5 kg on a frictionless table is attached to a hanging mass of 2 kg by a cord through a hole in the table. The 5 kg mass is following a circular path of radius 2 m. What speed must the 5 kg mass have to maintain this circular motion?

The 2kg mass supplies the centripetal for required to make the 5kg mass go around in the circle $W = m_1 g$ $F_r = -\frac{m_2 v^2}{r}$

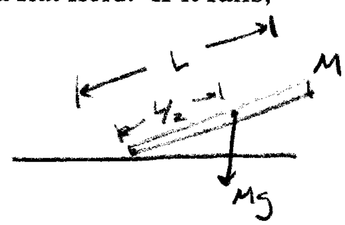


$|W| = |F_r| \Rightarrow m_1 g = \frac{m_2 v^2}{r}$ $\quad v^2 = \frac{m_1 g r}{m_2}$

$$V = \left[\frac{(2kg)(9.8m/s^2)2m}{5kg}\right]^{1/2} = 2.8m/s$$

Section II - Problems

II-1 (20 pts) A flag pole (a long thin rod) of length L stands vertically in a flat field. If it falls, what is the acceleration of the top of the pole as it hits the ground?
($I_{\text{rod about end}} = (1/3)ML^2$)



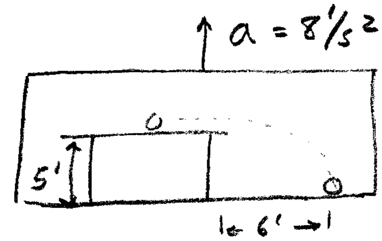
just as flag pole hits the ground, the weight is perpendicular to the pole $\Rightarrow \tau = Mg\frac{L}{2}$

$\tau = I\alpha$ $\quad \alpha = \frac{Mg\frac{L}{2}}{\frac{1}{3}ML^2}$

at end of pole $a = \alpha L = \frac{\cancel{M}g\frac{\cancel{L}}{2}}{\frac{1}{3}\cancel{M}\cancel{L^2}} \cdot \cancel{L} = \frac{3}{2}g = 14.7\,m/s^2$

Name (1 pt) ____________________________

II-2 (20 pts). A table is located in an elevator which is accelerating upward at 8 ft/s^2. A ball rolls horizontally off the edge of the table, which is 5.0 ft above the floor of the elevator. It strikes the floor at a point 6.0 ft horizontally away from the edge of the table. What was the speed of the ball at the instant it left the table?



If you are in an accelerating frame of reference relative to an inertial frame (i.e. the earth), you must add the acceleration of the frame to gravity

$\Rightarrow$ In the elevator: $g' = a + g = 40'/s^2$

Now work the projectile problem:

Vertical: $y - y_0 = \cancel{v_{0y}t}^{\,0} - \frac{1}{2} g' t^2$

$5' = +\frac{1}{2}(40'/s^2)t^2$

$t^2 = \frac{5'}{20'/s^2} \Rightarrow \boxed{t = \frac{1}{2}\ s \text{ when ball hits floor}}$

Horizontal:

$x - x_0 = v_{0x}t + \cancel{\frac{1}{2}a_x t^2}^{\,0}$

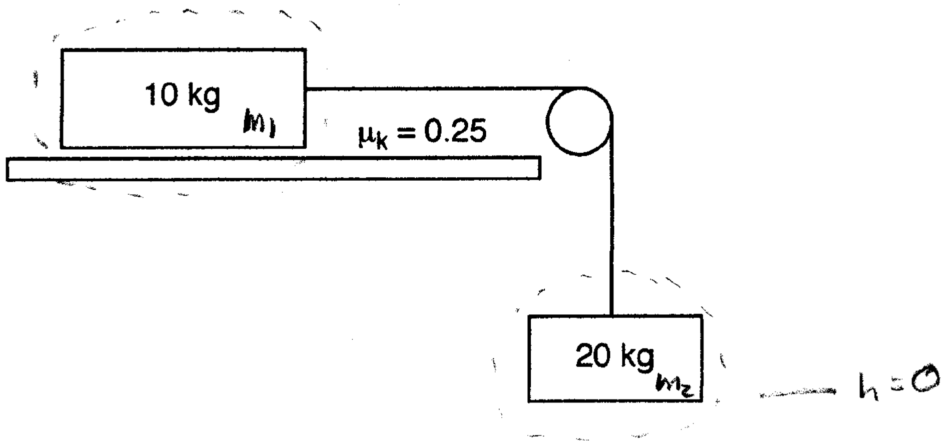
$6' = v_{0x}t \rightarrow \boxed{v_{0x} = \frac{6'}{\frac{1}{2}\ s} = 12'/s}$

because $v_{0y} = 0$

$\boxed{v = 12'/s}$

Name (1pt) ______________________________

II-3 (24 pts) Two blocks are connected by string and a massless pulley as shown in the figure. The block on the table has a mass of 10 kg and a coefficient of kinetic friction with the table of 0.25. The hanging block has a mass of 20 kg. If the blocks start from rest, what is their velocity after the blocks move 5m?



This is an "end point" problem ⇒ conserve energy

$E_i = \cancel{K_i}^{\,0} + \cancel{U_i}^{\,0}$ $\qquad E_f = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 - m_2 g h \qquad$ (h = d

$W = -f_k d = -\mu_k m_1 g d$

$E + W = E_f$

$0 - \mu_k m_1 g d = \frac{1}{2}(m_1 + m_2) v^2 - m_2 g d$

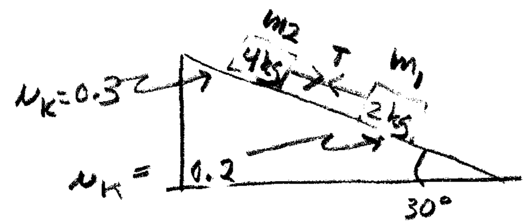
$$\frac{(m_2 - \mu_k m_1) g d}{\frac{1}{2}(m_1 + m_2)} = v^2$$

$$\frac{(20\,kg - 2.5\,kg)(9.8\,m/s^2)(5\,m)}{15\,kg} = v^2$$

$$\boxed{v = 7.6\,m/s}$$

Name (1 pt) ______________________________

II-4 (24 pts) Two masses, $m_1 = 2$ kg and $m_2 = 4$ kg are connected by a massless string and are sliding down a 30° incline plane, with m_2 ahead of m_1. The coefficient of kinetic friction between m_1 and the incline is 0.2 and between m_2 and the incline 0.3. What is the acceleration of the system?



Since the μ_K is smaller for the leading block

$\Rightarrow$ string is taut

$\Rightarrow$ can treat both blocks as a single system, T as an internal force

$$\sum F_{\text{along incline plane}} = m_2 g \sin\theta + m_1 g \sin\theta - \mu_{K_2} m_2 g \cos\theta - \mu_{K_1} m_1 g \cos\theta$$

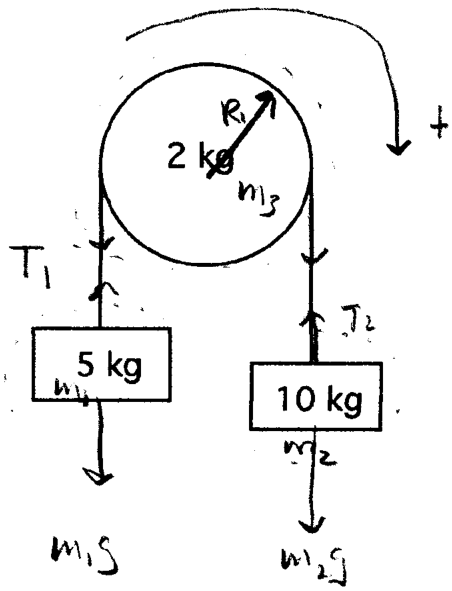
$$= 4\,\text{kg} \cdot 9.8\,\text{m/s}^2 (1/2) + 2\,\text{kg} \cdot 9.8\,\text{m/s}^2 (1/2) - 0.3 \cdot 4\,\text{kg}\ 9.8 (.87) - 0.2 \cdot 2\,\text{kg}\ 9.8 (.87)$$

$$= (19.6 + 9.8 - 10.2 - 3.4)\,\text{N} = 15.8\,\text{N}$$

$$F = ma \Rightarrow a = \frac{15.8\,\text{J}}{6\,\text{kg}} = 2.63\,\text{m/s}^2$$

Name (1pt) ______________________________

III-5 (24 pts) A mass of 5 kg and a mass of 10 kg are attached to a massless string and hung over a 2 kg pulley (assume that it has the shape of a solid disk). If the masses are released from rest, how long does it take the 10 kg mass to fall 10 m?
($I_{disk} = (1/2)MR^2$)



$m_2 g - T_2 = m_2 a$ $F = ma$

$T_1 - m_1 g = m_1 a$ $F = ma$

$T_2 R - T_1 R = \frac{1}{2} M R^2 \alpha$ $\tau = I\alpha$ $m_1 =$

$T_2 R - T_1 R = \frac{1}{2} M R^2 \frac{a}{R}$ $a = \alpha R$

$m_2 g - m_1 g = \left(m_1 + m_2 + \frac{M}{2}\right) a$

$$a = \frac{(10\,kg - 5\,kg)(9.8\,m/s^2)}{5\,kg + 10\,kg + 2\,kg/2} = 3.06\,m/s^2$$

$$y - y_0 = \cancel{v_{0y} t}^{\,0} + \frac{1}{2} a t^2$$

$$10\,m = \frac{1}{2}(3.06\,m/s^2)\, t^2$$

$$\boxed{t = 2.56\,s}$$