

Name (1 pt): _____

Recitation Instructor (1 pt): _____

There are four pages to this midterm. It is important that you write your name on each page and the name of your recitation instructor on the first page. Each name is worth one point.

Be sure to include the proper units in your answers.

Useful basic equations:

$$v = v_0 + at$$

$$x - x_0 = v_0 t + (1/2) at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = (1/2)(v_0 + v)t$$

$$x - x_0 = vt - (1/2) at^2$$

$$F = ma \text{ (Newton's second law)}$$

$$a = v^2/r \text{ (for uniform circular motion)}$$

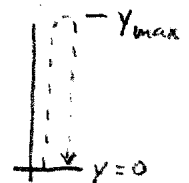
Problem 1. A ball is projected straight up from ground level at a speed of 64 ft/s.

a) (5pts) What is the maximum elevation that the ball will attain?

$$v^2 = v_0^2 - 2g(y - y_0)$$

$$0 = (64 \text{ ft/s})^2 - 2 \cdot 32 \text{ ft/s}^2 (y_{\text{max}} - 0) \text{ at top}$$

$$\boxed{y_{\text{max}} = 64'}$$



b) (5pts) How long will it take to reach that height?

$$v = v_0 - gt$$

$$0 = 64 \text{ ft/s} - 32 \text{ ft/s}^2 \cdot t$$

$$\boxed{t = 2 \text{ s}}$$

or

$$y - y_0 = \frac{1}{2}(v_0 + v)t$$

$$64' = \frac{1}{2}(64 \text{ ft/s} + 0)t$$

$$\boxed{t = 2 \text{ s}}$$

c) (5pts) At $t = 3$ seconds what will the speed and direction of the ball be?

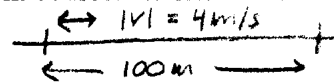
$$v = v_0 - gt$$

$$= 64 \text{ ft/s} - 32 \text{ ft/s}^2 \cdot 3$$

$$\boxed{v = -32 \text{ ft/s (i.e. downward)}}$$

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Problem 2. A runner is running back and forth along a 100 m course. When he turns around, he does so instantaneously. His speed is a constant 4 m/sec.



a) (5pts) What is his average velocity (including sign) over the time interval $t = 0 - 75$ seconds?

- at 4 m/s, in 75s total travel distance = $4 \text{ m/s} \cdot 75 \text{ s} = 300 \text{ m}$
- this will place him at far end of course \Rightarrow displacement = $(\Delta x) = 100 \text{ m}$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{+100 \text{ m}}{75 \text{ s}} = 1.3 \text{ m/s}$$

b) (5pts) What is his average acceleration (including sign) in the interval $t = 20 - 30$ seconds?

- at $t = 20 \text{ s}$, will have traveled to $+80 \text{ m} \Rightarrow v_1 = +4 \text{ m/s}$
- at $t = 30 \text{ s}$, will have traveled $120 \text{ m} \rightarrow$ back to $x = +80 \Rightarrow v_2 = -4 \text{ m/s}$

$$\bar{a} = \frac{v_2 - v_1}{\Delta t} = \frac{-4 \text{ m/s} - (+4 \text{ m/s})}{30 \text{ s} - 20 \text{ s}} = \frac{-8 \text{ m/s}}{10 \text{ s}} = -0.8 \text{ m/s}^2 = \bar{a}$$

Problem 3. A rabbit runs in a two dimensional coordinate system such that its position is given by $x = 5t^2 - 3t$ and $y = 2t - 7$, where t is in seconds and x and y are in meters.

a) (5pts) In unit vector notation, what is the rabbit's position vector at $t = 5 \text{ s}$?

$$\text{at } t = 5: x = (5 \cdot 5^2 - 3 \cdot 5) \text{ m} = 110 \text{ m}$$

$$y = (2 \cdot 5 - 7) \text{ m} = 3 \text{ m}$$

$$\vec{r} = [110\hat{i} + 3\hat{j}] \text{ m}$$

b) (5pts) In unit vector notation, what is the rabbit's velocity vector at $t = 5 \text{ s}$?

$$v_x = \frac{dx}{dt} = 10t - 3; \text{ at } t = 5: v_x = (10 \cdot 5 - 3) \text{ m/s} = 47 \text{ m/s}$$

$$v_y = \frac{dy}{dt} = 2; \text{ at } t = 5: v_y = 2 \text{ m/s}$$

$$\vec{v} = [47\hat{i} + 2\hat{j}] \text{ m/s}$$

c) (5pts) What is the rabbit's speed at $t = 5 \text{ s}$?

$$s = |\vec{v}| = (47^2 + 2^2)^{1/2} = 47.04 \text{ m/s}$$

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Problem 4. A plane is flying horizontally at 1000 ft/second at an altitude of 1024 ft. At $t = 0$ it releases a bomb.

a) (10 pts) How far will the bomb travel horizontally, before it hits the ground?

Y-motion to get time


$$Y - Y_0 = V_{0y}t - \frac{1}{2}gt^2$$

$$1024' = 0 - \frac{1}{2} \cdot 32' / s^2 \cdot t^2$$

$$t^2 = 64s^2, \quad t = 8 \text{ sec}$$

X-motion

$$x - x_0 = V_{0x}t = 1000' / s \cdot 8s = \boxed{8000' = \Delta x}$$

$$V_{0x} = 1000' / s$$


$$1024'$$

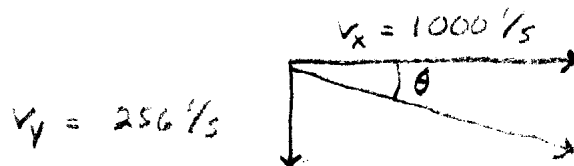
$$t = 0$$

b) (10 pts) With what speed will the bomb hit the ground?

$$V_y^2 = V_{0y}^2 - 2g(y - Y_0) = 0 - 2 \cdot 32' / s^2 \cdot 1024' \Rightarrow V_y = 256' / s$$

$$|V| = \sqrt{(256^2 + 1000^2)}^{1/2} / s = \boxed{1032' / s = S}$$

c) (10 pts) At what angle relative to the horizontal will the bomb hit the ground?
(Draw a diagram and unambiguously label this angle)

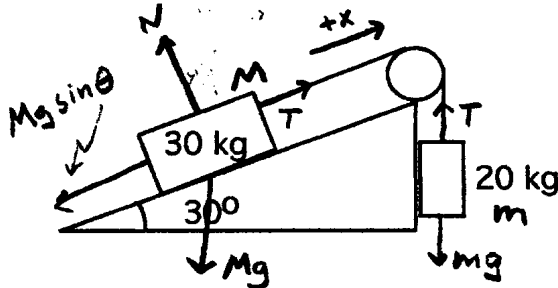


$$\tan^{-1} \theta = 0.256$$

$$\theta = 14^\circ$$

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Problem 5. A mass $m = 30 \text{ kg}$ is placed on a frictionless 30° incline. The block is connected to a cord that passes over a massless and frictionless pulley and is attached to a hanging 20 kg mass. At $t = 0$ the system is allowed to begin to move.



$$T - Mg \sin \theta = Ma$$

$$mg - T = ma$$

$$mg - Mg \sin \theta = (m + M)a$$

$$a = \frac{(m - M \sin \theta)g}{m + M} = \frac{[20 - 30(.5)]9.8}{20 + 30}$$

a) (10 pts) What is the acceleration of the system (magnitude AND direction)?

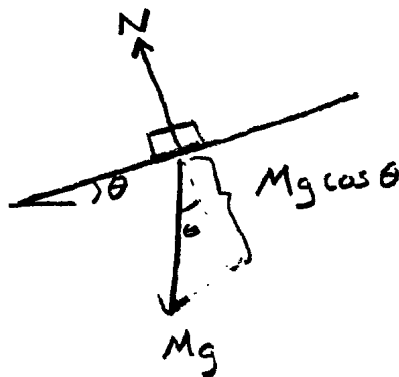
direction: up incline plane

$$a = \frac{9.8}{10} \text{ m/s}^2 = 0.98$$

b) (10 pts) What is the tension in the cord?

$$T = mg - ma = 20 \text{ kg} (9.8 \text{ m/s}^2 - 0.98 \text{ m/s}^2) = \boxed{177 \text{ N} = T}$$

c) (5 pts) What is the normal force on the sliding mass?



$$N - Mg \cos \theta = 0$$

$$N = Mg \cos \theta = [30 \cdot 9.8 \cdot 0.866] \text{ N}$$

$$\boxed{N = 255 \text{ N}}$$