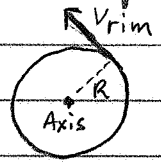
Chapter 13

30.)



$R = 0.040\,m$

$M = 0.10\,kg$

$K = 0.15\,J$

$V_{rim} = R\omega$

Find ω from $K = \frac{1}{2} I \omega^2 = 0.15\,J$

and find I from $I = \frac{1}{2} M R^2 = \frac{1}{2}(0.1)(0.04)^2 = 8.0 \times 10^{-5}\,kg\,m^2$

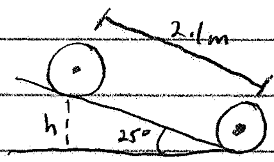
Solving for $\omega = \sqrt{\frac{2(0.15)}{I}} = \sqrt{\frac{0.30}{8 \times 10^{-5}}} = \boxed{61.237\ rad/s}$

32.) a.) $V_{cm} = R\omega = 20\,m/s \Rightarrow \omega = 20\,m/s / 0.3\,m = 66.7\,rad/s = 66.7\left(60/2\pi\right) = \boxed{637\,rpm}$

b.) $V_{top} = 2V_{cm} = 2(20\,m/s) = \boxed{40\ m/s}$

c.) $V_{bottom} = \boxed{0\ m/s}$

34.)



a.) $K_f = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} M V_{cm}^2 = Mgh \Rightarrow \frac{1}{2}\left(\frac{2}{5} M R^2\right)\omega^2 + \frac{1}{2} M (R\omega)^2 = Mgh$

$\Rightarrow \frac{7}{10} M R^2 \omega^2 = Mg(2.1\,m)\sin(25) \Rightarrow \omega = \sqrt{\frac{\frac{10}{7} g (2.1\,m)(\sin 25°)}{(0.04\,m)^2}} = \boxed{88.1\ \frac{rad}{s}}$

b.) $K_{total} = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} M v_{cm}^2 = \frac{7}{10} M R^2 \omega^2$

$K_{rot} = \frac{1}{2} I_{cm}\omega^2 = \frac{1}{2}\left(\frac{2}{5} M R^2\right)\omega^2 = \frac{1}{5} M R^2 \omega^2$

$\Rightarrow \frac{K_{rot}}{K_{total}} = \frac{\frac{1}{5} M R^2 \omega^2}{\frac{7}{10} M R^2 \omega^2} = \frac{1}{5} \cdot \frac{10}{7} = 0.286$

62.) a.) $\tau = I\alpha$ Find α using $\omega_1 = \omega_0 + \alpha(t_1 - t_0) = 2000\left(\frac{2\pi}{60}\right) = 0 + \alpha(3.0s)$

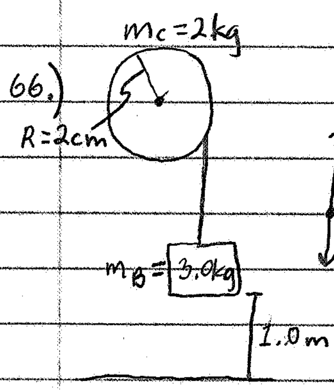
$\Rightarrow \alpha = \frac{200\pi}{9}\,rad/s^2$

$\tau = (2.5 \times 10^{-5}\,kg \cdot m^2)\left(\frac{200\pi}{9}\,rad/s^2\right) = 1.75 \times 10^{-3}\,N \cdot m$

b.) $\theta_1 = \theta_0 + \omega_0(t_1 - t_0) + \frac{1}{2}\alpha(t_1 - t_0)^2 = 0 + 0 + \frac{1}{2}\left(\frac{200\pi}{9}\,rad/s^2\right)(3.0\,s)^2$

$= 100\pi\,rad = 100\pi/2\pi$ revolutions $= \boxed{50\,rev}$

66.)



a.) Block: $T - w_B = m_B a_y$

Cylinder: $\tau = -TR = I\alpha = I\frac{a_y}{R} \Rightarrow T = -\frac{I a_y}{R^2}$

$$-\frac{I a_y}{R^2} - m_B g = m_B a_y \Rightarrow a_y = \frac{-m_B g}{(m_B + I/R^2)}$$

Since $I = m_c R^2$,

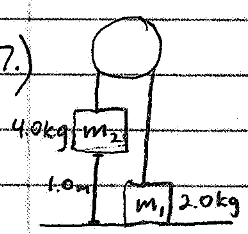
$$a_y = \frac{-m_B g}{(m_B + m_c)} = \frac{-(3.0kg)(9.8\ m/s^2)}{(3.0kg + 2.0kg)} = \boxed{-5.88\ m/s^2}$$

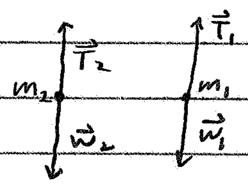
b.) $\frac{1}{2} m_B v_1^2 + \frac{1}{2} I \omega_1^2 + m_B g y_1 = \frac{1}{2} m_B v_0^2 + \frac{1}{2} I \omega_0^2 + m_B g y_0$

$\frac{1}{2} m_B v_1^2 + \frac{1}{2} I \omega_1^2 + 0 = 0 + 0 + m_B g y_0 \Rightarrow v_1^2 \left(\frac{m_B}{2} + \frac{m_c}{2}\right) = m_B g y_0$

$$v_1 = \sqrt{\frac{2 m_B g y_0}{m_B + m_c}} = \left(\frac{2(3\ kg)(9.8\ m/s^2)(1.0m)}{(3.0\ kg) + (2.0kg)}\right)^{1/2} = \boxed{3.43\ m/s}$$

67.)





$T_1 - w_1 = m_1 a_1 \qquad -w_2 + T_2 = m_2 a_2 \qquad T_2 R - T_1 R - 0.50 Nm = I\alpha$

Since $-a_2 = a_1 = a$, $I = \frac{1}{2} m_p R^2$, and $\alpha = a/R$, the above become

$T_1 - m_1 g = m_1 a \qquad m_2 g - T_2 = m_2 a \qquad T_2 - T_1 = \left(\frac{1}{2} m_p R^2\right)\left(\frac{a}{R}\right)\frac{1}{R} + \frac{0.5}{R} = \frac{m_p a}{2} + \frac{0.5}{0.06}$

Adding these three together yields:

$$(m_2 - m_1) g = a\left(m_1 + m_2 + \tfrac{1}{2} m_p\right) + 8.333 N$$

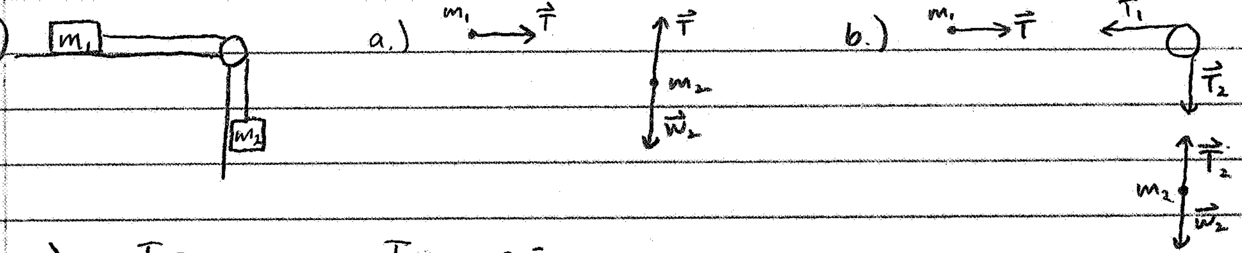
$$\Rightarrow a = \frac{(m_2 - m_1) g - 8.333}{m_1 + m_2 + \frac{1}{2} m_p} = \frac{(4.0 - 2.0)(9.8) - 8.333}{2.0 + 4.0 + (2.0/2)} = 1.610\ m/s^2$$

Now using kinematics: $y_1 = y_0 + v_0(t_1 - t_0) + \frac{1}{2} a_1 (t_1 - t_0)^2$

$$0 = 1 + 0 + \tfrac{1}{2}(-1.610)(t_1 - 0)^2$$

$$\Rightarrow t_1 = \sqrt{\frac{2(1.0)}{(1.610)}} = \boxed{1.11 s}$$

68.)



a.) $T = m_1 a_1 \qquad T - m_2 g = m_2 a_2$

Since $-a_2 = +a_1 = a$ by adding together the two above equations and solving for a:

$$\boxed{a = \frac{m_2 g}{m_1 + m_2} \quad \text{and thus} \quad T = m_1 a = \frac{m_1 m_2 g}{m_1 + m_2}}$$

b.) $T_1 = m_1 a \qquad T_1 R - T_2 R = -I\alpha \qquad \alpha$ is negative because pulley accelerates clockwise

$T_1 = m_1 a \qquad T_2 - T_1 = \frac{I}{R}\frac{a}{R} = \frac{aI}{R^2}$

Adding these two yields:

$$T_2 = a\left(m_1 + I/R^2\right)$$

Newton's second law for m_2: $T_2 - m_2 g = -m_2 a$. With this and the above equation for T_2, a can be solved for

$$a\left(m_1 + \frac{I}{R^2}\right) + m_2 a = m_2 g \Rightarrow a = \frac{m_2 g}{m_1 + m_2 + I/R^2} = \boxed{\frac{m_2 g}{m_1 + m_2 + \frac{1}{2} m_p}}$$

Now with a we can find T_1 and T_2

$$T_1 = m_1 a = \boxed{\frac{m_1 m_2 g}{m_1 + m_2 + \frac{1}{2} m_p}} \qquad T_2 = a\left(m_1 + I/R^2\right) = \frac{m_2 g}{(m_1 + m_2 + \frac{1}{2} m_p)}\left(m_1 + \frac{1}{2} m_p\right)$$

$$= \boxed{\frac{m_2\left(m_1 + \frac{1}{2} m_p\right) g}{m_1 + m_2 + \frac{1}{2} m_p}}$$

When $m = 0$ kg equations a, T_1, and T_2 become the same as in part a.)

84.) The center of mass of the ball moves in a circle of radius $R-r$. When it is at the top of the loop the diagram becomes. $\vec{n}\downarrow\downarrow\vec{w}$ and we want $\vec{n}=0$.

Thus, $mg+n=\frac{mv_1^2}{R-r}=mg+0$

$$v_1^2=g(R-r)$$

$$v_1^2=r^2\omega_1^2 \text{ since it's rolling}$$

Thus, $r^2\omega_1^2=g(R-r) \Rightarrow \omega_1^2=\frac{g(R-r)}{r^2}$

Conservation of energy:

$$\frac{1}{2}mv_1^2+\frac{1}{2}I\omega_1^2+mgy_1=mgh$$

Using $I=\frac{2}{5}mr^2$ the above becomes

$$\frac{1}{2}mg(R-r)+\frac{1}{2}\left(\frac{2}{5}\right)mr^2\left(\frac{g(R-r)}{r^2}\right)+mg2(R-r)=mgh$$

Thus, $\boxed{h=2.7(R-r)}$