Quiz 8

(a)

The easiest way to tackle this is using energy conservation,

$$E_i = E_f$$

$$KE_i + PE_i = KE_f + PE_F$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 + mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + mgh_f$$

Since the system starts from rest the initial kinetic energy is zero. Also, v and ω are related by $v = r\omega$. I here is the moment of inertia of the disk which is $1/2MR^2$. Thus, we have,

$$0 + 0 + mg(h_i - h_f) = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2$$

$$v_f^2 = \frac{mg(h_i - h_f)}{\frac{1}{2}m + \frac{1}{4}M}$$

$$v_f = \sqrt{\frac{mg(h_i - h_f)}{\frac{1}{2}m + \frac{1}{4}M}}$$

Plugging in the numbers, we have,

$$v_f = 5.715 \text{ m/s}$$

(b)

The acceleration of the 5 kg mass can be calculated by using kinematics,

$$v_f^2 = v_i^2 + 2a(h_f - h_i)$$

$$5.715^2 = 0 + 2a(2)$$

$$a = 8.167 \text{ m/s}^2.$$

(c)

The angular acceleration and linear acceleration are related by $a = R\alpha$. Thus, the angular acceleration of the disk is

$$\alpha = \frac{8.167}{0.1} = 81.67 \text{ rad/s}^2.$$