

HOMEWORK ASSIGNMENT # 3

*DUE: Friday, 7 March*

1. Consider the molecule dimethyl ether ( $\text{CH}_3\text{OCH}_3$ ), which possesses two internally rotating methyl tops, each with a potential given by

$$V = \frac{V_3}{2}(1 - \cos 3\phi).$$

- Determine the eigenvalues and eigenfunctions for the internal rotation in the limit of free rotation (i.e. each  $V_3=0$ ). You need not define the rotational constant  $F$ .
- Write down a suitable basis set for the actual internal rotation problem based on free rotor functions in terms of  $\phi_1=1,0,\dots,1$  and  $\phi_2 = 1,0,\dots,1$ . Derive the matrices that, upon diagonalization, solve the problem.
- What would you expect the degeneracy structure to be for each torsional eigenvalue? Use reasonable labels similar to the  $A$ (degeneracy =1) and  $E$ (degeneracy=2) labels for the single methyl case.
- Can you guess how the degeneracy pattern would change if there were a coupling term between the two rotors; e.g.

$$V = V_{12} \cos 3\phi_1 \cos 3\phi_2$$

2. Show that the selection rule for transitions between torsional substates in the single methyl rotor problem is  $A$ - $A$  and  $E$ - $E$  only. Hint: consider how the dipole moment changes with torsional angle  $\phi$  as the methyl rotates.

3. Derive the formula used in class for the probability of a transition from a lower level to an upper level

$$P_{u \leftarrow l} = |c_u(t)|^2$$

$$\text{where } c_u(t) = \frac{i}{\hbar} \int_0^t \exp(-i(E_u - E_l)t/\hbar) H_{lu} dt$$

via time-dependent perturbation theory.