Integrated Absorption Coefficients

$$I(\nu) = I_0(\nu)\, e^{-K_\nu \ell}$$

$$\text{Absorbed Intensity (erg/cm}^2\text{ s)} = \int [I_0(\nu) - I(\nu)]\, d\nu \cong I_0 \int (1 - e^{-K_\nu \ell})\, d\nu$$

assuming constant intensity

For weak absorption $K_\nu \ell \ll 1$: $1 - e^{-K_\nu \ell} \approx K_\nu \ell$

$$\text{Absorbed Intensity} \cong I_0 \ell \int K_\nu d\nu$$

Now, from theory: $\dfrac{dn_\ell}{dt} = -\underbrace{B_{u\leftarrow\ell}\, p(\nu_{u\ell})\, n_\ell}_{\text{molecules cm}^{-3}\text{ s}^{-1}}$

$$\text{Absorbed energy } (\text{cm}^{-3}\text{ s}^{-1}) = B_{u\leftarrow\ell}\, p(\nu_{u\ell})\, n_\ell\, h\nu_{u\ell}$$

$$\text{Absorbed energy in a cell of length } \ell \ (\text{cm}^{-2}\text{ s}^{-1}) = B_{u\leftarrow\ell}\, p(\nu_{u\ell})\, n_\ell\, h\nu_{u\ell}\, \ell$$

$$\therefore \underbrace{I_0 \ell \int K_\nu d\nu}_{\text{expt.}} = \underbrace{B_{u\leftarrow\ell}\, p(\nu_{u\ell})\, n_\ell}_{\text{theory}}\, h\nu_{u\ell}\, \ell$$

$$B_{u\leftarrow\ell} = \frac{2\pi}{3\hbar^2} |\mu|_{u\ell}|^2 \qquad I_0 = c\, p(\nu_{u\ell})$$

$$\Rightarrow \int K_\nu d\nu = \frac{8\pi^3 \nu_{u\ell}\, n_\ell}{3hc} |\mu|_{u\ell}|^2$$