Physics 829 Spring 2001
Dr. Herbst

MIDTERM EXAMINATION (In Class; Closed Book)
11 MAY
(100 POINTS)

(1) (30 points) Estimate the energy through first-order of perturbation theory for a hydrogen atom in its ground state with a finite spherical nucleus of radius $R \ll a_0$. You may assume that the nuclear charge resides on the surface of the nucleus.

$$\psi_{100} = \frac{1}{\sqrt{\pi a_0^{\,3}}}\exp(-r/a_0) \qquad E^0_{100} = -\frac{e^2}{2a_0} \qquad W = e^2\left(\frac{1}{r}-\frac{1}{R}\right);\ \ r \le R$$

What is the ratio of the first-order to the zeroth-order term if $R = 10^{-4}\ a_0$?

(2) (30 points) Use the WKB method to determine the energies of the states of a one-dimensional harmonic oscillator of charge q placed in a static electric field. (The answer is the same as the exact one.)

(3) (40 points) A system with angular momentum quantum number J=1 is maintained in a magnetic field such that the energy levels from lowest (M=-1) to highest (M=1) are separated by an amount $\Delta E_{1,0} = \Delta E_{0,-1} = \hbar\omega_0$. A second, rotating, magnetic field (with angular frequency ω) is applied at t=0 and turned off at τ, leading to the perturbation

$$W(t) = \omega_1[J_+\exp(-i\omega t) + J_-\exp(i\omega t)]$$

where J_+ and J_- are the raising and lowering operators. At t=0, the system is in the M=0 state.

a) (30 points) Determine at t=τ to first order (i) the state vector of the system and (ii) the probability that the system is in each of its three M states, as a function of ω. You must start with perturbation theory; do not use the formula for P_{fi} given on the equation page.

b) (10 points) Determine the second-order contribution to the population of the M=1 state.