SOME BASIC EQUATIONS

$H = T + V = P^2 / 2M + V(R)$ $\qquad \rho = |\psi><\psi|$

$H|\psi> = i\hbar \frac{\partial |\psi>}{\partial t}$ $\qquad H|\psi> = E|\psi>$

$\sum_i |u_i><u_i| = 1$ $\qquad \int d\alpha\, |\alpha><\alpha| = 1$

$P_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$ $\qquad X = i\hbar \frac{\partial}{\partial p_x}$

$[J_x, J_y] = i\hbar J_z$ $\qquad \mathrm{J}^2 \,|\, jm> = j(j+1)\hbar^2 \,|\, jm>$

$J_z \,|\, jm> = m\hbar \,|\, jm>$ $\qquad m = j, j-1, j-2, ..., -j$

$J_\pm = J_x \pm iJ_y$ $\qquad L_z = \frac{\hbar}{i}\frac{\partial}{\partial \phi}$

$${}^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2} r + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}(\sin\theta \frac{\partial}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

$L^2 = -\hbar^2\{\frac{\partial^2}{\partial\theta^2} + \frac{1}{\tan\theta}\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\}$ $\qquad [-\frac{\hbar^2}{2m}\frac{d^2}{dr^2} + \frac{\ell(\ell+1)\hbar^2}{2mr^2} + V(r)]u_{k,\ell}(r) = \frac{k^2\hbar^2}{2m}u_{k,\ell}(r)$

$Y_l^m(\theta,\vartheta) = N_l^m e^{im\vartheta} P_l^m(\theta);$ $\quad P_l^m$ - associated Legendre functions

$$\cos\theta Y_l^m = \sqrt{\frac{(l+m+1)(l-m+1)}{(2l+1)(2l+3)}}Y_{l+1}^m + \sqrt{\frac{(l+m)(l-m)}{(2l+1)(2l-1)}}Y_{l-1}^m$$

Some Spherical Harmonics:

$Y_0^0 = \sqrt{1/4\pi}$ $\qquad Y_1^{\pm1} = \mp\sqrt{3/8\pi}\sin\theta e^{\pm i\phi}$

$Y_1^0 = \sqrt{3/4\pi}\cos\theta$ $\qquad Y_2^{\pm1} = \mp\sqrt{15/8\pi}\sin\theta\cos\theta e^{\pm i\phi}$

$Y_2^0 = \sqrt{5/16\pi}(3\cos^2\theta - 1)$ $\qquad Y_2^{\pm2} = \sqrt{15/32\pi}\sin^2\theta e^{\pm 2i\phi}$

APPROXIMATION METHODS

$\int_a^b k\,dx = \pi(n + 1/2)$ $\quad$ $H \geq E_0$ $\quad$ $P_{tunn} = \exp\left(-2\int_{x_1}^{x_2} \kappa(x)\,dx\right)$

$E_n^2 = E_n^0 + W_{nn} + \sum_{p \neq n} \frac{|W_{np}|^2}{E_n^0 - E_p^0}$ $\quad$ $\lambda \langle \phi_p | 1 \rangle = \frac{1}{E_n^0 - E_p^0} \langle \phi_p | W | \phi_n \rangle ;\ p \neq n$

$H = \sum_{i=1}^{N} H_i^0 + \sum_{i<j} \frac{e^2}{r_{ij}}$ $\quad$ $H \approx \sum_i \left(H_i^0 + V_i^{eff}\right);\ V_i^{eff} = \sum_{j \neq i} \int \frac{|\phi_j(j)|^2}{r_{ij}} d\tau_j$

$\psi(1,2,3,...N) = \prod_{i=1}^{N} \phi_i(i)$

$i\hbar \frac{db_n(t)}{dt} = \sum_k e^{iw_{nk}t} W_{nk}(t) b_k(t)$ can be solved by successive approximation

$\Gamma = \frac{2\pi}{\hbar} |W_{fi}|^2 \rho(E)\,d\Omega$ $\quad$ $\rho(E) = \frac{2\omega^2}{(2\pi c)^3 \hbar} V$ $\quad$ $\Gamma = \frac{2\pi}{\hbar} |W'_{fi}|^2 \rho(E)\,d\Omega$

$\langle n+1 | a^+ | n \rangle = (n+1)^{1/2}$ $\quad$ $\langle n-1 | a | n \rangle = n^{1/2}$

$W(t) \quad -\frac{q}{m} P_z A_z = -\frac{q}{m} P_z \left\{ A_0 e^{i(ky - \omega t)} + A_0\, e^{-i(ky - \omega t)} \right\}$

$W_{DE}(t) = \frac{qE}{m\omega} P_z \sin \omega t$ $\quad$ $W_{DM}(t) = \frac{-qB}{2m} (L_x + 2S_x) \cos \omega t$

$W_{QE}(t) = \frac{-qE}{2mc} (YP_z + ZP_y) \cos \omega t$ $\quad$ $\mathbf{A} = \sum_\lambda \left(\frac{\hbar}{2\omega_\lambda} \right)^{1/2} \left[a_\lambda \mathbf{A}_\lambda + a_\lambda \mathbf{A}_\lambda \right] \sqrt{\frac{1}{\varepsilon_0 V}}$

$\mathbf{A}_\lambda = \mathbf{e}_\lambda e^{i\mathbf{k}_\lambda \cdot \mathbf{r}}$ $\quad$ $b_m = \langle \phi_m^{(1)} | \phi_i^{(0)} \rangle$

$P_{fi}(\omega, t) = \frac{|W_{fi}|^2}{4\hbar^2} \frac{\sin^2\left[(\omega - \omega_{fi}) t/2\right]}{\left[(\omega - \omega_{fi})/2\right]^2}$ $\quad$ $S = \frac{1}{N!} \sum_\alpha P_\alpha$ $\quad$ $A = \frac{1}{N!} \sum_\alpha \varepsilon_\alpha P_\alpha$

(if ω=0, then remove "4" from denominator)