Dirac Delta Function

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(k)\, e^{ikx}\, dk \qquad (1a)$$

$$g(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx \qquad (1b)$$

With x' as a dummy variable, substitute (1b) into (1a) to get

$$f(x) = \int_{-\infty}^{\infty} dx'\, f(x')\, \delta(x-x')$$

after rearrangement, where

$$\delta(x-x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik(x-x')}\, dk$$ is the

Fourier representation of the Dirac Delta Function. A more simple representation is

$$\delta(x) = \lim_{\epsilon \to 0} \begin{cases} \frac{1}{\epsilon} & -\epsilon/2 < x < \frac{\epsilon}{2} \\ 0 & |x| > \epsilon/2 \end{cases}$$

Some properties of δ: $\int_{-\infty}^{\infty} \delta(x-x')\, dx = \underline{1}$

$$\delta(x) = \delta(-x)$$

[redacted]