Physics 829

Spring 2000
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Answers To Final Examination

(1) $1s \rightarrow n\ell m$ (absorption)

semiclassical treatment $W \rightarrow W_{DE}(t) = \frac{q\mathcal{E}}{m\omega} P_z \sin\omega t$

for each resonant frequency:

$$P_{1s\rightarrow n\ell m}(\omega, t) = \frac{|W_{n\ell m, 1s}|^2}{4\hbar^2} \underbrace{\frac{\sin^2[(\omega - \omega_{n\ell m;1s})t/2]}{[(\omega - \omega_{n\ell m;1s})/2]^2}}_{t^2 \text{ at resonance}}$$

$$W_{n\ell m;1s} = \frac{qE}{m\omega} \langle n\ell m | P_z | 1s \rangle$$

$$[z, \mathcal{H}^0] = [z, P_z^2/2m] = i\hbar P_z/m$$

$$\langle n\ell m | [z, \mathcal{H}^0] | 1s \rangle = \langle n\ell m | z\mathcal{H}^0 - \mathcal{H}^0 z | 1s \rangle$$
$$= (E_1 - E_n) \langle n\ell m | z | 1s \rangle$$
$$= -\hbar\omega_{n1} \langle n\ell m | z | 1s \rangle = \frac{i\hbar}{m} \langle n\ell m | P_z | 1s \rangle$$

$$\therefore \langle n\ell m | P_z | 1s \rangle = i m \omega_{n1} \langle n\ell m | z | 1s \rangle$$

$$W_{n\ell m, 1s} = qEi\frac{\omega_{n1}}{\omega} \underbrace{\langle n\ell m | z | 1s \rangle}_{\text{investigate selection rules}}$$

$z = r\cos\theta$

$$\langle n\ell m | z | 1s \rangle = \int_{r=0}^{\infty} \underbrace{R_{n\ell}^* \, r R_{10}}_{\substack{\text{no } n \text{ selection} \\ \text{rule}}} r^2 dr \int_{\Omega} Y_\ell^{m*} \cos\theta \, Y_0^0 \, d\Omega$$

From equation sheet: $\cos\theta \, Y_\ell^m = (\quad) Y_{\ell+1}^m + (\quad) Y_{\ell-1}^m$

$\therefore$ In Ω integral: $m=0$, $\ell = 1$ (p)

Possible Final States: $n > 1$ $\ell = 1$ $m = 0$

Quantum Treatment

$$W = -\frac{q}{m} P_z A_z \stackrel{\sim}{=} -\frac{q}{m} P_z \sqrt{\frac{\hbar}{2\omega_\lambda}\frac{1}{\varepsilon_0 V}}\,(a_\lambda + a_\lambda^+)$$

time-independent

This W is for one monochromatic resonant mode.

Use $P_{n\ell m \leftarrow 1s} = \frac{1}{\hbar^2}\Big| -\frac{q}{m} \langle n\ell m | P_z | 1s \rangle$
(field loses ω_λ)

$$\times \sqrt{\frac{\hbar}{2\omega_\lambda}\frac{1}{\varepsilon_0 V}} \langle n_\lambda - 1 | a_\lambda | n_\lambda \rangle \Big|^2 t^2$$

Next evaluate $\langle n\ell m | P_z | 1s \rangle$ as in semiclassical treatment

(2) Write $|\psi\rangle = \sum_n a_n |\varphi_n\rangle$ $\quad |\varphi_n\rangle$ actual eigenkets of $\mathcal{H}$.

$$\mathcal{H}|\psi\rangle = \sum a_n \mathcal{H}|\varphi_n\rangle$$
$$= \sum a_n E_n |\varphi_n\rangle$$

$$\langle \psi|\mathcal{H}|\psi\rangle = \sum_m \sum_n a_m^* E_n a_n \underbrace{\langle \varphi_m|\varphi_n\rangle}_{\delta_{mn}}$$

$$= \sum_n |a_n|^2 E_n \geq E_0 \sum_n |a_n|^2$$

$$\langle \psi|\psi\rangle = \sum_m \sum_n a_m^* a_n \langle \varphi_m|\varphi_n\rangle = \sum_n |a_n|^2$$

$$\therefore \langle \psi|\mathcal{H}|\psi\rangle \geq E_0 \langle \psi|\psi\rangle$$

$$\frac{\langle \psi|\mathcal{H}|\psi\rangle}{\langle \psi|\psi\rangle} \geq E_0$$

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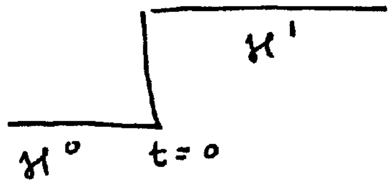

$$\mathcal{H}' = \mathcal{H}^0 - \vec{\mu}\cdot\vec{E}$$

$$\mathcal{H}' = \mathcal{H}^0 - \frac{1}{2} q \frac{Z^2}{a} E_z$$

(a) Use sudden approximation

$t<0$ eigenkets $|\varphi_i^0\rangle$ = standard H.O. kets

$$\langle z|\varphi_i^0\rangle = \varphi_i^0(z)$$

$$t>0 \quad \mathcal{H}' = T + \frac{1}{2}\mu\omega^2 z^2 - \frac{1}{2} q \frac{E_z}{a} z^2$$

$$\mathcal{H}' = T + \frac{1}{2}\left(\mu\omega^2 - \frac{q E_z}{a}\right) z^2$$

$$\mathcal{H}' = T + \frac{1}{2}\mu\Big(\underbrace{\omega^2 - \frac{q E_z}{\mu a}}_{\omega'^2}\Big) z^2$$

assume >0.

$\therefore |\varphi_m^{(1)}\rangle$ kets of H.O. with reduced frequency.

$$\beta^2 = \mu\omega'/\hbar$$

$$b_m = \langle \varphi_m^{(1)} | \varphi_0^{(0)}\rangle$$

$$|\Psi(t>0)\rangle = \sum_m b_m e^{-iE_m^{(1)} t/\hbar} |\varphi_m^{(1)}\rangle$$

$$\langle E\rangle = \sum |b_n|^2 E_n \quad \text{constant}$$

(b) $$b_0 = \langle \varphi_0^{(1)} | \varphi_0^{(0)}\rangle = \int_{-\infty}^{\infty} \varphi_0^{(1)*}\varphi_0^0 \, dz$$

$$b_0 = \sqrt{\frac{\alpha}{\pi^{1/2}}\frac{\beta}{\pi^{1/2}}} \underbrace{\int_{-\infty}^{\infty} e^{-[\alpha^2+\beta^2]z^2/2} \, dz}_{\sqrt{\pi} \; 1/\sqrt{(\alpha^2+\beta^2)/2}}$$

$$\therefore\ b_0 = \sqrt{\frac{2\alpha\beta}{\alpha^2+\beta^2}}$$

$$\text{Prob}\ (0\to 0) = |b_0|^2 = \frac{2\alpha\beta}{\alpha^2+\beta^2} = \frac{2\,\omega^{1/2}\,\omega'^{1/2}}{\omega+\omega'}$$

where $\omega' = \omega\left[1 - \frac{qE_z}{\mu\omega^2 a}\right]^{1/2}$

(4) <u>He</u>

(a) $1s^2 \quad |\psi_A\rangle = |1:1s\rangle|2:1s\rangle\,|0,0\rangle \qquad {}^1S$

$|0,0\rangle = \frac{1}{2}\{|1:+\rangle|2:-\rangle - |1:-\rangle|2:+\rangle\}$

$L=0$

$1s2s \quad |\psi_A\rangle = \frac{1}{\sqrt{2}}\{|1:1s\rangle|2:2s\rangle - |1:2s\rangle|2:1s\rangle\}\,|1,M_s\rangle \quad {}^3S$

$L=0$

$$|1,M_s\rangle = \left\{\begin{array}{ll} |1:+\rangle|2:+\rangle & ;\ M_s=1 \\ 1/\sqrt{2}\,(|1:+\rangle|2:-\rangle + |1:-\rangle|2:+\rangle) & ;\ M_s=0 \\ |1:-\rangle|2:-\rangle & ;\ M_s=-1 \end{array}\right\}$$

$|\psi_A\rangle = \frac{1}{\sqrt{2}}\{|1:1s\rangle|2:2s\rangle + |1:2s\rangle|2:1s\rangle\}\,|0,0\rangle \quad {}^1S$

$1s2p \quad |\psi_A\rangle = \frac{1}{\sqrt{2}}\{|1:1s\rangle|2:2p_x\rangle - |1:2p_x\rangle|2:1s\rangle\}\,|1,M_s\rangle$

$L=1$ $\qquad {}^3P$

$x = 1,0,-1$

$|\psi_A\rangle = \frac{1}{\sqrt{2}}\{|1:1s\rangle|2:2p_x\rangle + |1:2p_x\rangle|2:1s\rangle\}\,|0,0\rangle$

${}^1P$

$1s^2({}^1S) < 1s2s({}^3S) < 1s2s({}^1S) < 1s2p({}^3P) < 1s2p({}^1P)$

(b) $S_{TOTAL} = 2, 0 \; (S)$ $\qquad$ $S_{TOTAL} = 1 \; (A)$

$|\Psi\rangle = |\Psi_s\rangle$

$$1s^2 \quad |\Psi_s\rangle = |1:1s\rangle|2:1s\rangle \begin{Bmatrix} |2, M_s\rangle & {}^5S \\ |0,0\rangle & {}^1S \end{Bmatrix}$$

$L=0$

$1s2s$

$$|\Psi_s\rangle = \frac{1}{\sqrt{2}}\left\{|1:1s\rangle|2:2s\rangle - |1:2s\rangle|2:1s\rangle\right\} |1, M_s\rangle \quad {}^3S$$

$L=0$

$$|\Psi_s\rangle = \frac{1}{\sqrt{2}}\left\{|1:1s\rangle|2:2s\rangle + |1:2s\rangle|2:1s\rangle\right\} \begin{Bmatrix} |2, M_s\rangle & {}^5S \\ |0,0\rangle & {}^1S \end{Bmatrix}$$

$$\underbrace{1s^2\,({}^5S) = 1s^2\,({}^1S)}_{\text{same spatial function}} < 1s2s\,({}^3S) < 1s2s\,({}^5S = {}^1S)$$

$S = 1/2$ $\qquad$ $\underline{\mathcal{H}^0}$ $\qquad$ $S = 1$