Physics 829 Spring 2001

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Answers To Midterm

(1) $\dot{\bar{e}}$ $\quad w = e^2\left(\frac{1}{r} - \frac{1}{R}\right) \quad r \le R << a_0 \qquad \psi_{100} = (\pi a_0^3)^{-1/2} e^{-r/a_0}$

$$\langle \psi_{100} | w | \psi_{100} \rangle = \frac{1}{\pi a_0^3} e^2 \, 4\pi \int_{r=0}^{R} r^2 dr \left\{ e^{-2r/a_0} \left(\frac{1}{r} - \frac{1}{R} \right) \right\}$$

$$\langle \psi_{100} | w | \psi_{100} \rangle = \frac{4e^2}{a_0^3} \left\{ \int_{r=0}^{R} e^{-2r/a_0} r\,dr - \frac{1}{R} \int_0^R e^{-2r/a_0} r^2 dr \right\}$$

$e^{-2r/a_0} \approx 1$ over range $0 \le r \le R$

$$\langle \psi_{100} | w | \psi_{100} \rangle = \frac{4e^2}{a_0^3} \left\{ \int_{r=0}^{R} r\,dr - \frac{1}{R} \int_0^R r^2 dr \right\}$$

$$= \frac{4e^2}{a_0^3} \left\{ R^2/2 - R^2/3 \right\} = \frac{2e^2 R^2}{3a_0^3}$$

$$E = E^0_{100} + \langle \psi_{100} | w | \psi_{100} \rangle = \frac{-e^2}{2a_0} + \frac{e^2}{a_0} \left[\frac{2R^2}{3a_0^2} \right]$$

$$E = \frac{-e^2}{2a_0} \left\{ 1 - \frac{4}{3} \left(\frac{R}{a_0} \right)^2 \right\}$$

$$\frac{\langle \psi_{100} | w | \psi_{100} \rangle}{E^0} = \frac{\lambda E^{(1)}}{E^0} = -\frac{4}{3} \left(\frac{R}{a_0} \right)^2 = -\frac{4}{3} \times 10^{-8}$$

(2) $\displaystyle\int_a^b k\,dx = \pi(n+1/2) \qquad V(x) = \frac{1}{2} m\omega^2 x^2 - q\mathcal{E} x$

$$V(x) = \frac{m\omega^2}{2}\left[x^2 - \frac{2q\mathcal{E}x}{m\omega^2} \right] = \frac{m\omega^2}{2}\left\{ x - \frac{q\mathcal{E}}{m\omega^2} \right\}^2 - \frac{q^2\mathcal{E}^2}{2m\omega^2}$$

Let $y = x - q\mathcal{E}/m\omega^2 \qquad V(y) = \frac{m\omega^2}{2} y^2 - \frac{q^2\mathcal{E}^2}{2m\omega^2}$

turning points: $E = V(y) \qquad \frac{m\omega^2}{2} y^2 = E + q^2\mathcal{E}^2/2m\omega^2$

$$\frac{1}{\hbar} \int_{y_1}^{y_2} \sqrt{2m(E - V(y))}\, dy = \pi(n+1/2)$$

$$\sqrt{2m}\int_{Y_1}^{Y_2}\sqrt{E+\frac{q^2\mathcal{E}^2}{2m\omega^2}-\frac{m\omega^2}{2}y^2}\,dy = \frac{h}{2}(n+1/2)$$

$$a^2 \equiv E+\frac{q^2\mathcal{E}^2}{2m\omega^2} \Rightarrow \sqrt{2m}\int_{Y_1}^{Y_2}\sqrt{a^2-\frac{m\omega^2}{2}y^2}\,dy = \frac{h}{2}(n+1/2)$$

let $z = \sqrt{\dfrac{m\omega^2}{2}}\,y$

$$\frac{h}{2}(n+1/2) = \sqrt{2m}\int_{z_1}^{z_2}\sqrt{a^2-z^2}\,dz\sqrt{\frac{2}{m\omega^2}}$$

$$\frac{h}{2}(n+1/2) = \frac{2}{\omega}\cdot 2\int_{0}^{z_2}\sqrt{a^2-z^2}\,dz$$

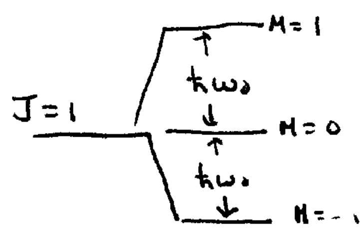
$$z_2^2 = E+q^2\mathcal{E}^2/2m\omega^2 = a^2 \qquad z_2 = a > 0$$

$$\frac{h}{2}(n+1/2) = \frac{1}{2}\frac{4}{\omega}\left\{ z\sqrt{a^2-z^2} + a^2\sin^{-1}\frac{z}{|a|}\,\Big|_0^a \right\}$$

$$= \frac{2}{\omega}\left\{a^2\sin^{-1}1\right\} = \frac{a^2\pi}{\omega} = \frac{\pi}{\omega}\left\{E+\frac{q^2\mathcal{E}^2}{2m\omega^2}\right\}$$

$$\therefore\quad E+\frac{q^2\mathcal{E}^2}{2m\omega^2} = \frac{h\omega}{2\pi}(n+1/2)$$

$$E_n = \hbar\omega(n+1/2) - q^2\mathcal{E}^2/2m\omega^2$$

③ 

$$W = \omega_1\left[J_+e^{-i\omega t} + J_-e^{i\omega t}\right]$$

$$|\Psi(0)\rangle = |M=0\rangle$$

$$|\Psi(t)\rangle = \sum_M b_M(t)\exp(-iE_mt/\hbar)\,|M\rangle \qquad i\hbar\frac{db_n}{dt} = \lambda\sum_k e^{i\omega_{nk}t}\,\hat{W}_{nk}(t)b_k$$

$$b_k = b_k^0 + \lambda b_k^1 + \ldots$$

zeroth order: $\quad i\hbar\dfrac{db_n^0}{dt} = 0 \qquad b_n^0(t) = b_n^0(0)$

$$b_1^0 = b_{-1}^0 = 0 \qquad b_0^0 = \underline{1}$$

1^{ST} order :
$$i\hbar \frac{db_n^{(1)}}{dt} = \sum_k e^{i\omega_{nk} t} \hat{W}_{nk}(t)\, b_k^{0}(t)$$

$k = 0$ only non-zero $b_M^{(0)}$
$$\therefore\ i\hbar \lambda \frac{d_M^{(1)}}{dt} = e^{i\omega_{M0} t} W_{M0}(t)$$

$$W_{M0}(t) = \omega_1 \langle M|J_+|0\rangle e^{-i\omega t} + \omega_1 \langle M|J_-|0\rangle e^{i\omega t}$$
$$\langle M|J_+|0\rangle = \hbar\sqrt{2}\ \delta_{M1} \qquad \langle M|J_-|0\rangle = \hbar\sqrt{2}\ \delta_{M-1}$$

$\therefore$ only $M = \pm 1$ can be reached in 1^{ST} order.
$$W_{M0}(t) = \hbar\omega_1\sqrt{2}\left\{ \delta_{M1}\, e^{-i\omega t} + \delta_{M-1}\, e^{i\omega t} \right\}$$
$$\omega_{M0} = \omega_0\ (M=1)\ ;\ -\omega_0\ (M=-1)$$

$$M=1 \quad i\hbar\lambda \frac{db_1^{(1)}}{dt} = e^{i\omega_0 t}\hbar\omega_1\sqrt{2}\, e^{-i\omega t} = \sqrt{2}\,\hbar\omega_1\, e^{i(\omega_0-\omega)t}$$
$$M=-1 \quad i\hbar\lambda \frac{db_{-1}^{(1)}}{dt} = e^{-i\omega_0 t}\hbar\omega_1\sqrt{2}\, e^{i\omega t} = \sqrt{2}\,\hbar\omega_1\, e^{i(\omega-\omega_0)t}$$

$$\lambda b_1^{(1)}(\tau) = \frac{\sqrt{2}\omega_1}{i}\int_0^\tau e^{i(\omega_0-\omega)t}\,dt = \frac{\sqrt{2}\,\omega_1}{i}\ \frac{e^{i(\omega_0-\omega)\tau}-1}{i(\omega_0-\omega)}$$
$$b_1(\tau) = b_1^{(0)} + \lambda b_1^{(1)} = \frac{\sqrt{2}\,\omega_1}{\omega-\omega_0}\quad e^{i(\omega_0-\omega)\tau} - 1$$
$$\lambda b_{-1}^{(1)}(\tau) = \frac{\sqrt{2}\,\hbar\omega_1}{i\hbar}\int_0^\tau e^{i(\omega-\omega_0)t}\,dt = \frac{\sqrt{2}\,\omega_1}{i}\ \frac{e^{i(\omega-\omega_0)\tau}-1}{i(\omega-\omega_0)}$$
$$b_{-1}(\tau) = \frac{\sqrt{2}\,\omega_1}{\omega_0-\omega}\quad e^{i(\omega-\omega_0)\tau} - 1$$
$$b_0(\tau) \approx b_0^{(0)} = 1 \qquad |\psi(t)\rangle = \sum b_M(\tau)\, e^{-iE_m\tau/\hbar}\, |M\rangle$$
$$P_{M=1} = |b_1|^2 = \frac{2\omega_1^2}{(\omega-\omega_0)^2}\,\left|e^{i(\omega_0-\omega)\tau/2}\right|^2\,\Big|\underbrace{e^{i(\omega_0-\omega)\tau/2} - e^{-i(\omega_0-\omega)\tau/2}}_{2i\sin(\omega_0-\omega)\tau/2}\Big|^2$$

$$P_{M=1} = 2w_1^2 \frac{\sin^2 (\omega-\omega_0)\tau/2}{\left(\frac{\omega-\omega_0}{2}\right)^2}$$

Similarly:

$$P_{M=-1} = \frac{2w_1^2}{(\omega_0-\omega)^2} \left| e^{i(\omega-\omega_0)\tau/2} \right|^2 \left| \underbrace{e^{i(\omega-\omega_0)\tau/2} - e^{-i(\omega-\omega_0)\tau/2}}_{2i \sin (\omega-\omega_0)\tau/2} \right|^2$$

$$P_{M=-1} = 2w_1^2 \frac{\sin^2 (\omega-\omega_0)\tau/2}{\left(\frac{\omega-\omega_0}{2}\right)^2} = P_{M=1}$$

$$P_{M=0} = |b_0|^2 \approx 1$$

<u>2nd Order</u>

$$i\hbar \frac{db_1^{(2)}}{dt} \lambda^2 = \sum_{M=-1,0,1} e^{i\omega_{1M} t} \lambda \hat{W}_{1M}(t)\, b_M^{(1)}(t) \lambda$$

$$= e^{i\omega_{11} t} \underbrace{W_{11}(t)}_{0} \lambda b_1^{(1)}(t)$$

$$+ e^{i\omega_{10} t} W_{10}(t) \lambda \underbrace{b_0^{(1)}}_{0}(t)$$

$$+ e^{i\omega_{1-1} t} \underbrace{W_{1-1}}_{0}(t) \lambda b_{-1}^{(1)}(t) = 0$$

$b_1^{(2)}(\tau) = b_1^{(2)}(0) = 0$ no contribution