"Motivation" of Schrödinger Equation

Start with the classical wave equation:

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$$

Try $\Psi(x,t) = e^{-i\omega t}\,\psi(x)$

This leads to

$$\psi''(x) = -\omega^2/v^2 \; \psi(x)$$

$$v = \lambda\nu = \frac{\lambda}{2\pi}\,\omega = \hbar\,\omega/p \quad \text{via de Broglie}$$

$$\omega^2/v^2 = p^2/\hbar^2 = 2m(E-V)/\hbar^2$$

$$\therefore \quad \psi''(x) = -\frac{2m(E-V)}{\hbar^2}\,\psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi(x) = E\psi(x) \qquad (1)$$

$$\Psi(x,t) = e^{-i\omega t}\,\psi(x) = e^{-iEt/\hbar}\,\psi($$

$$i\hbar\frac{\partial \Psi}{\partial t} = E\,e^{-iEt/\hbar}\,\psi(x)$$

Multiply each side of (1) by $e^{-iEt/\hbar}$

Then:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

$x)$