

Dr. Herbst

Answers To Set 4

$$1. \quad V = -\frac{C}{R^4} \quad C = \frac{e^2 d}{2} \quad V_{\text{eff}} = -\frac{C}{R^4} + E_T b^2 / R^2$$

$$\frac{dV_{\text{eff}}}{dR} = 0 = 4CR^{-5} - 2E_T b^2 R^{-3} \Rightarrow 4CR^{-5} = 2E_T b^2 R^{-3}$$

$$\therefore 4C = 2E_T b^2 R^2 \Rightarrow R_{\text{max}}^2 = 2C / E_T b^2$$

$$\text{Orbiting Condition: } V_{\text{eff}}^{\text{max}}(R_{\text{max}}) = E_T = -\frac{C}{(2C/E_T b^2)^2} + \frac{E_T b^2}{2C/E_T b^2}$$

$$E_T = -C E_T^2 b_{\text{orb}}^4 / 4C^2 + \frac{E_T^2 b_{\text{orb}}^4}{2C} = +E_T^2 b_{\text{orb}}^4 / 4C$$

$$\therefore b_{\text{orb}}^4 = 4C / E_T \quad b_{\text{orb}}^2 = 2(C/E_T)^{1/2}$$

$$\sigma_e = \pi b_{\text{orb}}^2 = 2\pi (C/E_T)^{1/2} = 2\pi \left[ \frac{e^2 d}{2 \cdot \frac{1}{2} \mu v^2} \right]^{1/2}$$

$$\sigma_e = 2\pi e / v \sqrt{\alpha / \mu}$$

$$k_L = v \sigma_e = 2\pi e \sqrt{\alpha / \mu}$$

$$2. \quad \text{H}_3^+ \quad T = 10\text{K} \quad n = 10^4 \text{cm}^{-3}$$

$$a) \quad k_L(\text{H}_3^+ - \text{H}_2) \quad \alpha(\text{H}_2) = 0.8023 \times 10^{-24} \text{cm}^3$$

$$\frac{1}{\mu} = \frac{1}{m(\text{H}_3^+)} + \frac{1}{m(\text{H}_2)} = 0.82686 \Rightarrow \mu = 1.20939 / N_A \text{ gm}$$

$$(m(^1\text{H}) = 1.007825 \text{amu}) \quad \therefore k_L = 1.91 \times 10^{-9} \text{cm}^3 \text{s}^{-1}$$

$$e = 4.803 \times 10^{-10} \text{esu}$$

$$N_A = 6.022169 \times 10^{23}$$

$$\tau_{\text{H}_2} = \frac{1}{k_L n} = 5.24 \times 10^4 \text{s} = 14.5 \text{hr}$$

$$b) \quad k_L(\text{H}_3^+ - \text{CO}) \quad \alpha(\text{CO}) = 1.95 \times 10^{-24} \text{cm}^3$$

$$m(^{12}\text{C}) = 12.0000 \quad m(^{16}\text{O}) = 15.99491 \text{amu}$$

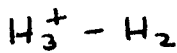
$$\frac{1}{\mu} = \frac{1}{m(\text{CO})} + \frac{1}{m(\text{H}_3^+)} = 0.36647 \quad \mu = 2.729/\text{NA} \text{ gm}$$

$$k_L(\text{H}_3^+ - \text{CO}) = 1.98 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1}$$

$$\tau_{\text{CO}} = \frac{1}{k_L [\text{CO}]} \quad [\text{CO}]/n \sim 10^{-4} \\ \therefore [\text{CO}] \sim 1.0 \text{ cm}^{-3}$$

$$\tau_{\text{CO}} = \frac{1}{k_L} = 5.05 \times 10^8 \text{ s} = 16.0 \text{ yr}$$

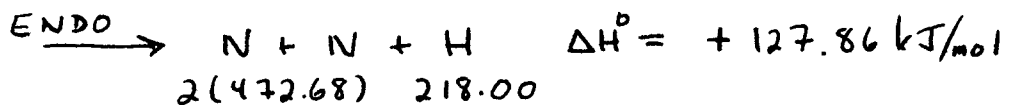
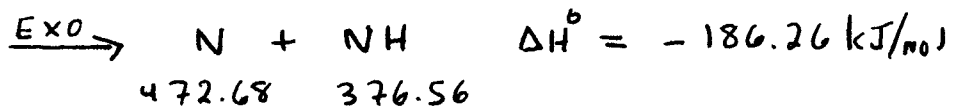
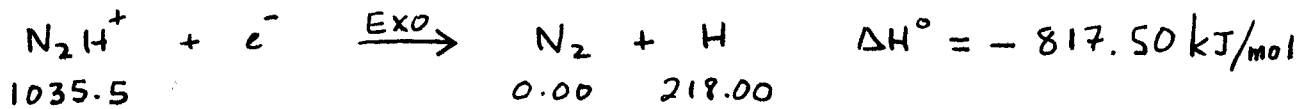
$$c) \text{ MFP} = v \tau \quad v = \sqrt{\frac{8kT}{\pi\mu}} = 4.18 \times 10^4 \text{ cm s}^{-1}$$



$$\text{MFP} = 4.18 \times 10^4 \text{ cm s}^{-1} (5.24 \times 10^4 \text{ s}) \\ = 2.19 \times 10^9 \text{ cm} = 2.19 \times 10^4 \text{ km}$$



<u>Heats of Formation:</u>	<u>Species</u>	<u><math>\Delta H_f^\circ</math> (kJ mol<sup>-1</sup>)</u>
	$\text{HN}_2^+$	1035.5
	H	218.00
	N	472.68
	$\text{N}_2$	0.00
	NH	376.56



Of the two exothermic channels, chemical intuition argues that  $\text{N}_2 + \text{H}$  dominates, given the strong N-N chemical bond.

Nevertheless, according to Dappert et al.

[reference: *Astrophys J.* 609, 459 (2004)],

the branching ratios are as follows:

$$N_2 + H \quad 36\% \quad NH + N \quad 64\%$$

Science is often non-intuitive! Last month, the results were withdrawn!

$$4. \quad H^+ + HCN \rightarrow \text{Products} \quad \alpha(HCN) = 2.5 \times 10^{-24} \text{ cm}^3$$

$$m(^{14}N) = 14.00307 \text{ amu}$$

$$\mu = 0.9716/N_A \text{ gm}$$

$$\mu_D = \mu_D(HCN) = 2.984 \text{ D} = 2.984 \times 10^{-18} \text{ esu-cm}$$

$$k_L = 2\pi e \sqrt{\alpha/\mu} = 3.76 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1}$$

$$x = \frac{\mu_D}{\sqrt{2\alpha k_B T}} \quad k_B = 1.3807 \times 10^{-16} \text{ erg/K}$$

$$x = 113.6/\sqrt{T} \quad x(10 \text{ K}) = 35.9$$

$$k_{LD}/k_L = 1 + \frac{2x}{\sqrt{\pi}} = 41.5$$

$$k_{TS}/k_L = 0.62 + 0.4767x = 17.7$$

$$k_{LD}(10 \text{ K}) = 1.56 \times 10^{-7} \text{ cm}^3 \text{ s}^{-1}$$

$$k_{TS}(10 \text{ K}) = 6.66 \times 10^{-8} \text{ cm}^3 \text{ s}^{-1}$$

$$\text{Expt. data at 300K: } k_{\text{meas}} = 1.1 \times 10^{-8} \text{ cm}^3 \text{ s}^{-1}$$

$$k_{TS}(300 \text{ K}) = 1.4 \times 10^{-8} \text{ cm}^3 \text{ s}^{-1}$$