C/P 740

Winter Quarter 2008
Dr. Herbst

Answers To Assignment # 2

1. $N_H = 1.8 \times 10^{21} A_V \qquad A_V \approx \sigma_{gr} N_{gr} \qquad (Q \sim 1)$

$\therefore \; N_H = 1.8 \times 10^{21} \sigma_{gr} N_{gr}$

$N_{gr}/N_H = \dfrac{1}{1.8 \times 10^{21} \sigma_{gr}} \qquad \sigma_{gr} = \pi a^2 \qquad a = 0.1\mu = 10^{-5}\,\text{cm}$

$\sigma_{gr} = \pi \times 10^{-10}\,\text{cm}^2$

$\dfrac{N_{gr}}{N_H} = \dfrac{n_{gr}}{n_H} = 1.8 \times 10^{-12}$

Mass of a dust particle $= \rho V = \rho \frac{4}{3} \pi a^3 = 1.05 \times 10^{-14}$ gm

$\rho = 2.5\,\text{gm/cm}^3$

Mass H $= 1/N_A = 1.66 \times 10^{-24}$ gm

$\dfrac{\text{Mass of Dust}}{\text{Mass of H atoms}} = (1.8 \times 10^{-12}) \dfrac{1.05 \times 10^{-14}\,\text{gm}}{1.66 \times 10^{-24}\,\text{gm}} = 0.0114$

$\dfrac{\text{Mass of Carbon}}{\text{Mass of Hydrogen}} = \dfrac{12}{1} \times 2 \times 10^{-4} = 2.4 \times 10^{-3}$

only about 21% of what is needed.

2. $V_{dust} = \frac{4}{3} \pi a^3 = 4.19 \times 10^{-15}\,\text{cm}^3$

Assume each atom occupies a sphere of radius $1\,\text{Å} = 10^{-8}$ cm

$V_{atom} = \frac{4}{3} \pi r^3 = 4.19 \times 10^{-24}\,\text{cm}^3$

$N\text{ atoms} = \dfrac{4.19 \times 10^{-15}}{4.19 \times 10^{-24}} = 10^9$ atoms

$\text{Area}_{dust} = 4\pi a^2 = 1.26 \times 10^{-9}\,\text{cm}^2$

$A_{atom} = \pi r^2 = 3.14 \times 10^{-16}\,\text{cm}^2$

$N\text{ atoms on surface} = \dfrac{\text{Area of dust}}{\text{Area of atom}} = 4 \times 10^6$

3. $\lambda_{red} \cong 700 \text{ nm} = 0.700\ \mu$

$\tau_\lambda \propto \frac{1}{\lambda}$ since $Q_\lambda \propto 1/\lambda$

$\frac{\tau_{30\mu}}{\tau_{0.7\mu}} = \frac{0.7\mu}{30\mu} = 0.0233 \qquad \tau_{0.7\mu} = 10$

$\Rightarrow \tau_{30\mu} = 0.233$

$I_{30\mu}(L) = I_{30\mu}(0)\, e^{-0.233} = I_{30\mu}(0)\,(0.792)$

% through cloud = 79.2 %

4. CO $\quad \nu(1\text{-}0) \cong 2B_e \quad \nu(2\text{-}1) \cong 4B_e \quad \nu(3\text{-}2) \cong 6B_e$

$B_e = \frac{h}{8\pi^2 \mu R_e^2}$

(a)

$h = 6.6260755 \times 10^{-27}$ erg-s

$N_A = 6.0221367 \times 10^{23}$ amu/gm

$R_e = 1.12819 \times 10^{-8}$ cm

$m(^{12}C) = 12.0000 \qquad m(^{16}O) = 15.99491$

$\frac{1}{\mu} = \frac{1}{m(C)} + \frac{1}{m(O)} \Rightarrow 6.856208 = \mu$ (amu)

$\mu(\text{gm}) = 1.138501 \times 10^{-23}$ gm

$\therefore\ B_e = 5.791202 \times 10^{10}$ Hz $= 57.91202$ GHz

$\nu(J=1-0) = 115.824$ GHz (actual = 115.271 GHz)

$\nu(J=2-1) = 231.648$ GHz (actual = 230.538 GHz)

$\nu(J=3-2) = 347.472$ GHz (actual = 345.795 GHz)

(b) $P_J \propto (2J+1) \exp\left[-E_J/kT\right]$

$E_J = hB_e J(J+1) \qquad E_J/k = \frac{hB_e}{k} J(J+1)$

$k = 1.3806503 \times 10^{-16}$ erg/K $\qquad \frac{hB_e}{k} = 2.77934$ K

$E_J/kT \overset{T=10K}{=} 0.27793$

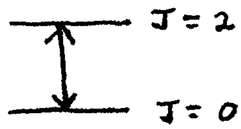
$P_{J=0} \propto 1 \cdot \exp(-0) = 1.0$

$P_{J=1} \propto 3 \cdot \exp(-0.27793 \times 1 \times 2) = 1.72 \leftarrow$

$P_{J=2} \propto 5 \cdot \exp(-0.27793 \times 2 \times 3) = 0.94$

5. Diffuse Clouds: electronic absorption spectrum, since a permanent dipole is not required. (UV)

Dense Clouds: weak quadrupolar transitions, for which $\Delta J = \pm 2$.



6. $\tilde{\nu}_P(J) = \tilde{\omega}_e - 2\tilde{B}_e J \qquad \tilde{\nu}_R(J) = \tilde{\omega}_e + 2\tilde{B}_e(J+1)$

a) $\tilde{\omega}_e = 2169.81\ cm^{-1} \qquad \tilde{B}_e = B_e/c = 1.9317\ cm^{-1}$

$v = 1 \leftrightarrow 0$

P Branch

$J-1 \leftarrow J$	$\tilde{\nu}\ (cm^{-1})$
$0 \leftarrow 1$	2165.95
$1 \leftarrow 2$	2162.08
$2 \leftarrow 3$	2158.22

R Branch

$J+1 \leftarrow J$	$\tilde{\nu}\ (cm^{-1})$
$1 \leftarrow 0$	2173.67
$2 \leftarrow 1$	2177.54
$3 \leftarrow 2$	2181.40

(b) Ignoring rotation: $E_v = h\omega_e(v + 1/2)$

Overtone Transition: $v = v' \leftarrow v = 0$

$$E_{v'} - E_0 = h\omega_e[v'] = h\nu = hc/\lambda$$

Visible $\lambda = 700\ nm = 7.00 \times 10^{-5}\ cm \qquad c = 2.9979 \times 10^{10}\ cm\,s^{-1}$
(red)

$$\omega_e v' = c/\lambda \qquad v' = \frac{c}{\lambda \omega_e} = \frac{1}{\lambda \tilde{\omega}_e} = 6.58$$

v' must be discrete

$\therefore v' = 7$ in visible $(7 \leftarrow 0)$

$(\lambda = 6583\ \text{Å})$

7. Star: black body 5000 K Dust: black body 300 K

a) $\lambda_{max} T = 3.00 \times 10^6$ nm-K

Star: $\lambda_{max} = 600$ nm (orange-red)

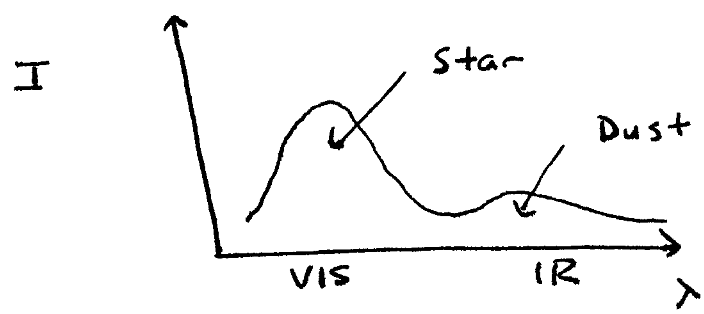
Dust: $\lambda_{max} = 1.00 \times 10^4$ nm $= 1.00 \times 10^{-3}$ cm

$\tilde{\nu}_{max} = 1000$ cm^{-1} (IR)

b) total intensity

$I = \sigma T^4$ $\sigma = 5.6704 \times 10^{-8}$ W/m^{2}K^4

Star: $I = 3.54 \times 10^7 \frac{W}{m^2}$ Dust: $I = 4.59 \times 10^2 \frac{W}{m^2}$



Of course, the stellar radiation will be affected by the foreground gas & dust.

Dust: extinction, reddening, spectroscopic absorption (IR)

Gas: IR absorption (vib-rot)