Physics 8820 **Homework 5** **Oct. 8**

(1) (Loudon, 2nd ed, problem 6.6)

Consider a pure state in which two photons are excited, one in each of two modes whose wavevectors, $\vec{k}_1$ and $\vec{k}_2$, are parallel to the z-axis with corresponding frequencies, ω_1 and ω_2. Show that the degree of first order coherence is:

$$g^{(1)}(\tau) = \{\omega_1 \exp(-i\, \omega_1\tau) + \omega_2 \exp(-i\, \omega_2\tau)\} \,/\, (\omega_1 + \omega_2)$$

where τ is the usual: $\tau = t_2 - t_1 - (z_2 - z_1)/c$. Compare to my solution for homework problem 2.3b.

(2) Result (d) from the notes, p61b, shows that the population in the excited state, |2>, oscillates as sine-squared if the laser is on resonance ($\Delta = 0$). In other words, the electron goes between being in the ground state to being in the excited state and back, forever. This unrealistic result came about because we did not account for randomizing events like spontaneous emission or collisions. However, for many experiments the frequency Ω of this oscillation, called the Rabi rate, is high enough that several or many oscillations occur before dephasing becomes important.

(a) What is the lowest value of Ωt ($t \geq 0$) that will put the atom in a superposition with equal populations in the ground and excited states?

(In the obscure parlance of atomic physics, a laser pulse that does this is called a "pi over 2 pulse". A laser pulse that exactly inverts the atom (exchanging populations in |1> and |2>) is called a "pi pulse".)

(b) What is the phase difference between the ground and excited states after the pulse in part (a)?

(3) (Fox) Here you'll show that (classically) $g^{(2)}(0) \geq g^{(2)}(\tau)$. Combined with homework problem 4.1, this completes our argument that the effects we are exploring in lecture are non-classical.

(a) Show that: $\sum_{i=1}^{N}[I(t_i)I(t_i+\tau)] \leq \frac{1}{2}\sum_{i=1}^{N}[I(t_i)^2 + I(t_i+\tau)^2]$

(b) For stationary light, $\langle I(t)^2\rangle = \langle I(t+\tau)^2\rangle$. Using the same definition for the <…> given in problem 4.1, this can be written as $\sum_{i=1}^{N} I(t_i)^2 = \sum_{i=1}^{N} I(t_i+\tau)^2$. Thus, the result in (a) can be written: $\sum_{i=1}^{N}[I(t_i)I(t_i+\tau)] \leq \sum_{i=1}^{N} I(t_i)^2$. Now show $g^{(2)}(0) \geq g^{(2)}(\tau)$.

(4) (Fox) The probability that an atom raised to its excited state at time t = 0 will decay in the interval from t to t + dt is: $dP(t) = 1/\tau_R \exp(-t/\tau_R)\, dt$ (similar to our expression for collisions). The probability for a decay in time T is found by integrating.

(a) Suppose a two-level atom is in a laser beam such that, if it is in the ground state, it is rapidly promoted to the excited state. If the excited state has decay time, τ_R, what is the probability of getting two decays in time T? (Assume the light does not affect the atom while it is in the excited state.)

(b) Suppose an HBT experiment uses photon counting detectors with a response time of τ_D. This means two events hitting each detector with a time delay less than τ_D would be observed as simultaneous. Estimate the value of $g^{(2)}(0)$ that would be expected for the atom in part (a).

(5) (Fox) A source emits a regular train of pulses, each containing exactly two photons. What value of $g^{(2)}(0)$ would be expected?

(6) What is the role of the scalers (pulse counters) in the Kimble experiment discussed in class? (I'm looking for a clear sentence or two. An essay isn't needed.)