Stability of Stable Cavities ("Dynamically and Mechanically Stable Resonators" S.S.3)

We want to satisfy three constraints:

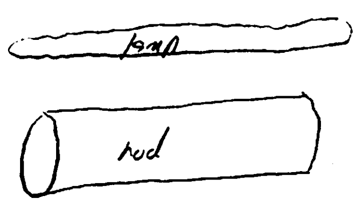
① Stable cavity (supports a mode as defined previously - in particular a Gaussian mode)

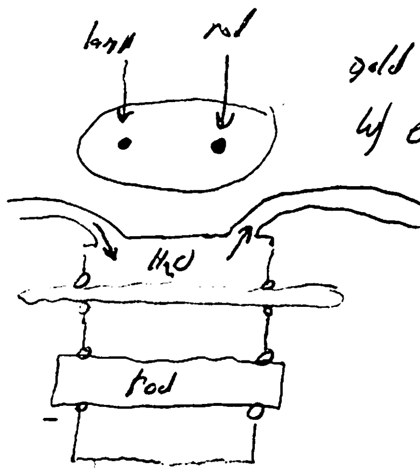
② There will be some constraint(s) on spot size. At a minimum, at least one imposed by the gain medium

③ Robustness - against length and tilts, vibrations (and intensity)

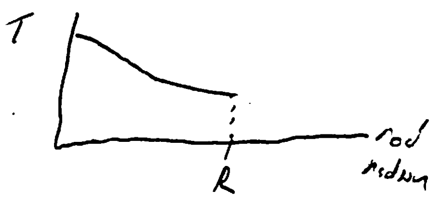
The text chooses to consider a specific cause of instability: thermal lensing and pump fluctuations.

This is a very common issue for high power solid state lasers.

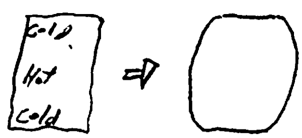




gold coated chamber w/ elliptical cross-section or diffuser

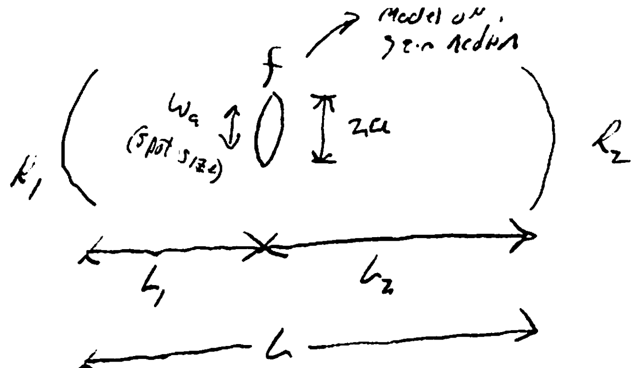


$n = n(T)$
and usually,
n increases w/ T.
(Also thermal expansion effects)



= a lens

If the pump power or cooling fluctuates, so will the lensing.



(#1) and assume we have $\left(\frac{A+D}{2}\right)^2 \leq 1$ $\boxed{77}$

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$$

(#3) We want the mode to be insensitive to changes in f.
Let the spot size at the active medium be W_a.
One way to require this is to demand: $\frac{dW_a}{d(1/f)} = 0$

Dynamic stability

$\frac{1}{f} \equiv$ dioptic power

[This is certainly not the only requirement you could imagine, but it's probably the most important. If W_a changes, I_a changes and thus your ability to saturate the transition.]

Thermal steering if misaligned through rod

(#2) $W_a \approx \frac{2a}{\pi}$ turns out to be a reasonable constraint.

This fills the medium as much as possible w/o diffraction losses

What do we expect?

We require $-1 \leq \frac{A+D}{2} \leq 1$ and $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2A_1D_1-1 & 2B_1D_1 \\ 2A_1C_1 & 2A_1D_1-1 \end{bmatrix}$

so $-1 \leq 2A_1D_1 - 1 \leq 1$

$0 \leq A_1D_1 \leq 1$

Moreover, ABCD has to be of the form:

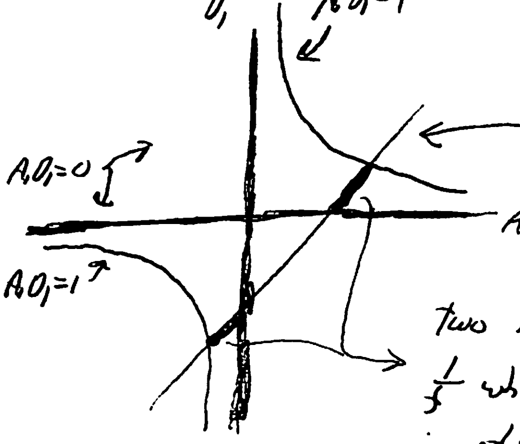
$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}\begin{pmatrix} i & j \\ k & l \end{pmatrix}$$

$A_1(1/f)$, $D_1(1/f)$ are linear in $1/f$

$A_1 = a\frac{1}{f} + b \Rightarrow \frac{1}{f} = \frac{A-b}{a}$

$D_1 = c\frac{1}{f} + d$

$\Rightarrow D_1 = \frac{c}{a}A_1 + d - \frac{bc}{a}$



two ranges in $\frac{1}{f}$ where the cavity is stable and dynamically stable

The width in $\frac{1}{f}$ of the two stable zones is the same

$$A_1 Q_1 = 1$$

$$A_1 = m Q_1 + b$$

$$A_1 = m \frac{1}{A_1} + b$$

$$A_1^2 - A_1 b - m = 0$$

$$A_1 = \frac{b \pm \sqrt{b^2 + 4m}}{2}$$

Length of positive segment (ℓ_+)

$$\ell_+^2 = (\Delta A_1)^2 + (\Delta Q_1)^2$$

$$= \left(\frac{b + \sqrt{b^2+4m}}{2} - b \right)^2 + \left(\frac{2}{b + \sqrt{b^2+4m}} \right)^2$$

Length of negative segment (ℓ_-)

$$\ell_-^2 = \left(\frac{b - \sqrt{b^2+4m}}{2} \right)^2 + \left(\frac{2}{b - \sqrt{b^2+4m}} - \left(-\frac{b}{m}\right) \right)^2$$

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In[17]:= Lp := ((b + Sqrt[b^2 + 4 m]) / 2 - b)^2 + (2 / (b + Sqrt[b^2 + 4 m]))^2

In[18]:= Lm := ((b - Sqrt[b^2 + 4 m]) / 2)^2 + (2 / (b - Sqrt[b^2 + 4 m]) + b / m)^2

In[19]:= Lp - Lm
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$$\text{Out[19]=} -\frac{1}{4}\left(b - \sqrt{b^2 + 4m}\right)^2 + \frac{4}{\left(b + \sqrt{b^2 + 4m}\right)^2} - \left(\frac{b}{m} + \frac{2}{b - \sqrt{b^2 + 4m}}\right)^2 + \left(-b + \frac{1}{2}\left(b + \sqrt{b^2 + 4m}\right)\right)^2$$

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In[20]:= Simplify[%]

Out[20]= 0
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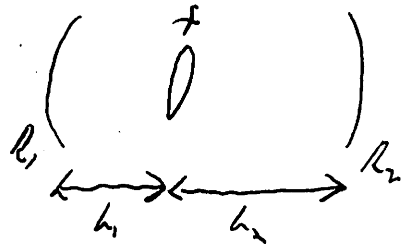
So $\ell_+ = \ell_-$, thus the amount by which $\frac{1}{f}$ can vary in each stable regime is the same.

Cavity Stability Condition



$$\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{R_2} & 1 \end{pmatrix}\begin{pmatrix} 1 & L_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}\begin{pmatrix} 1 & L_1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{R_1} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \frac{L_2}{f} - \frac{L_2 + L_1\left(1 - L_2/f\right)}{R_1} & L_2 + L_1\left(1 - L_2/f\right) \\ -\frac{1 - L_2/R_2}{f} - \frac{1 + L_1\left(-\frac{1 - L_2/R_2}{f} - \frac{1}{R_2}\right) - \frac{L_2}{R_2}}{R_1} - \frac{1}{R_2} & 1 + L_1\left(-\frac{1 - L_2/R_2}{f} - \frac{1}{R_2}\right) - \frac{L_2}{R_2} \end{pmatrix}$$

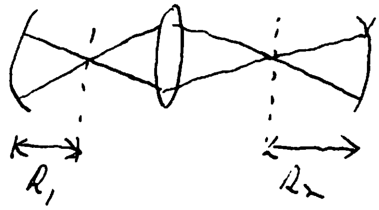
Check: get rid of the lens $\Rightarrow f \to \infty$

$$\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{L_2 + L_1}{R_1} & L_1 + L_2 \\ -\frac{1}{R_1} + \frac{L_1 + L_2}{R_1 R_2} - \frac{1}{R_2} & 1 - \frac{L_1 + L_2}{R_2} \end{pmatrix} = \begin{pmatrix} 1 - \frac{L}{R_1} & L \\ -\frac{1}{R_1} + \frac{L}{R_1 R_2} - \frac{1}{R_2} & 1 - \frac{L}{R_2} \end{pmatrix} \checkmark$$

$$-1 \le \frac{A+D}{2} \le 1 \Rightarrow A = D = 2A_1 D_1 - 1 \Rightarrow 0 \le A_1 D_1 \le 1$$

$$A_1 D_1 = \frac{\left[f(L_1 + L_2 - R_1) + L_2(-L_1 + R_1)\right]\left[f(L_1 + L_2 - R_2) + L_1(-L_2 + R_2)\right]}{f^2 R_1 R_2}$$

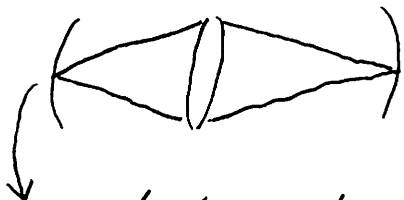
What is the nature of the two stable zones?



$$\frac{1}{f} = \frac{1}{L_1 - R_1} + \frac{1}{L_2 - R_2} \Rightarrow A_1 = \frac{(L_1 - R_1)R_2}{R_1(L_2 - R_2)} > 0$$

$$D_1 = \frac{R_1(L_2 - R_2)}{(L_1 - R_1)R_2} > 0$$

$$A_1 D_1 = 1$$



curved mirror acts like a flat if all the rays hit the same point

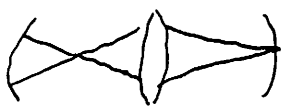
$$\frac{1}{f} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$L_1 = R_1 \quad L_2 = R_2$$

$$\Rightarrow A_1 = -\frac{L_2}{L_1} < 0 \qquad A_1 D_1 = 1$$

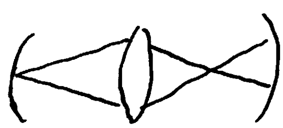
$$D_1 = -\frac{L_1}{L_2} < 0$$

And mix-n-match:



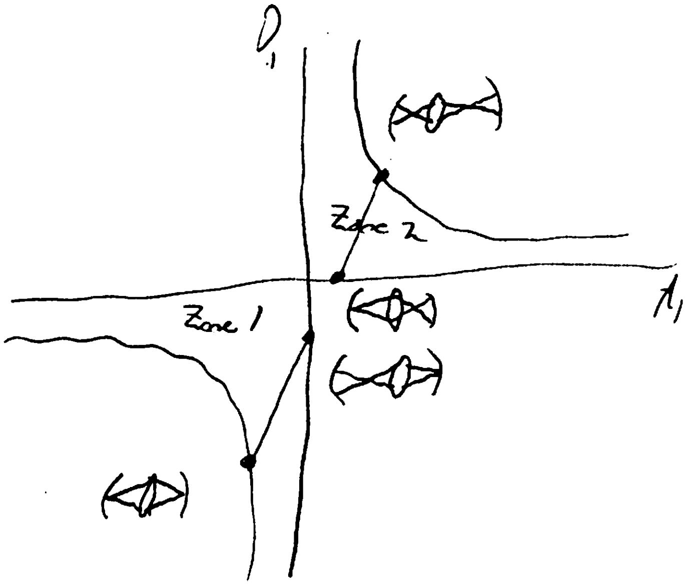
$$\frac{1}{f} = \frac{1}{L_1 - R_1} + \frac{1}{R_2} \Rightarrow A_1 = 0 \qquad A_1 D_1 = 0$$

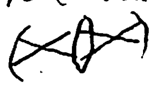
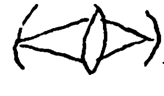
$$L_2 = R_2 \qquad D_1 = -\frac{L_1}{L_2}$$

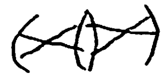


$$\frac{1}{f} = \frac{1}{R_1} + \frac{1}{L_2 - R_2} \qquad A_1 = -\frac{L_2}{L_1} \qquad A_1 D_1 = 0$$

$$L_1 = R_1 \qquad D_1 = 0$$



For fixed R_1 & R_2 & L_1 & L_2  requires shorter f than .

(or  has bigger $1/f$.)

<u>Thus, Zone 2 comes from harder pumping than Zone 1.</u>

At the (imaginary) plane mirror #1:

$$q_1 = i\sqrt{-\frac{B_1 D_1}{A_1 C_1}} \Rightarrow \frac{1}{q_1} = -i\sqrt{-\frac{A_1 C_1}{B_1 D_1}}$$

At the (real) mirror #1:

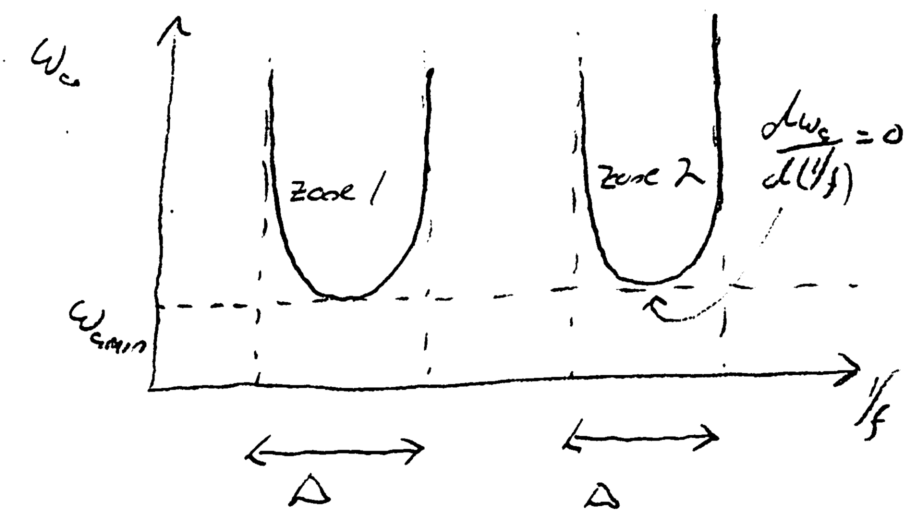
$$\frac{1}{q_1} = \frac{1}{R_1} - i\sqrt{-\frac{A_1 C_1}{B_1 D_1}} \qquad \left(\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{\pi w^2}\right)$$

At the active medium

$$q_a = q_1 + L_1 \qquad q_{out} = \frac{A q_{in} + B}{C q_{in} + D} \quad \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{sin} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$

and

$$w_a = \sqrt{-\frac{\lambda}{\pi \, Im(1/q_a)}}$$

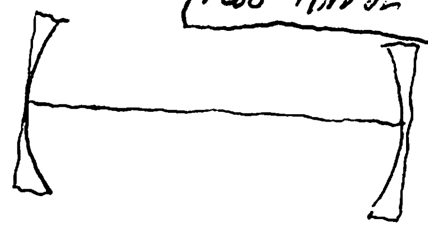


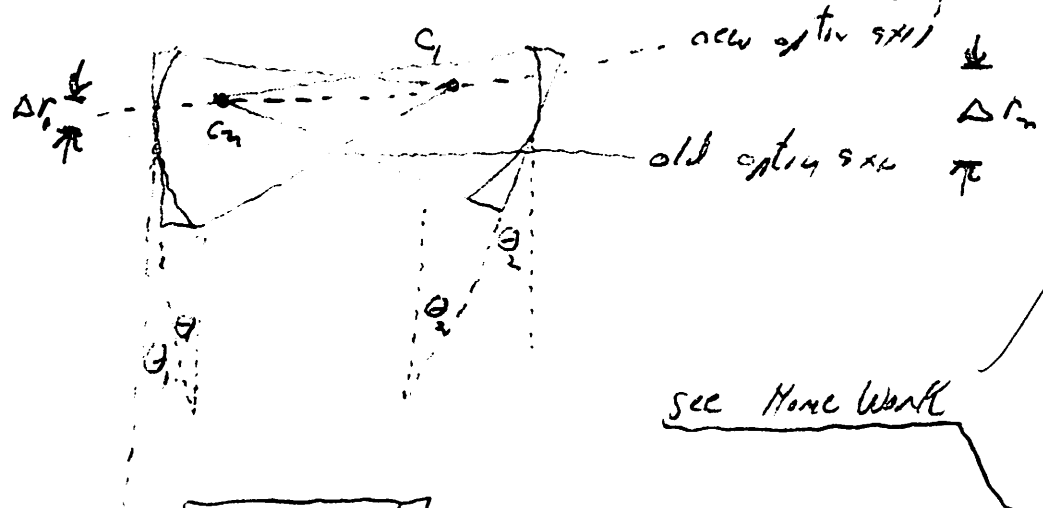
$$\frac{\pi w_{amin}^2}{\lambda}\Delta = 2$$

Adjust R_1, R_2, L_1, L_2 so $w_{amin} \approx \frac{2a}{\pi}$ if you can.

Sensitivity to misalignment

$\boxed{\text{Two mirror Cavity (see Siegman Ch A)}}$





$$\Delta r_1 = f(\theta_1, \theta_2)$$

$$\Delta r_2 = f(\theta_1, \theta_2)$$

see Home Work

* Find Δr_1 and Δr_2. (Assume θ_1 and θ_2 small. Δr_2 can be found from Δr_1 by swapping indices.)
* Rewrite in terms of $g_1 g_2$
* Find $\frac{d\,\Delta r_i}{d\theta_i}$.
* Can also find Gauss-axis transverse movement of mode

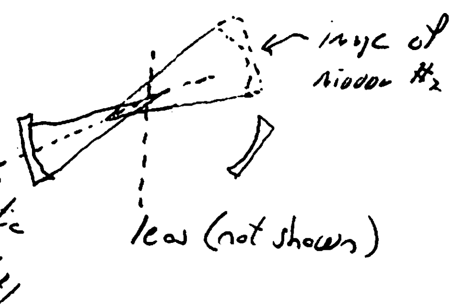
$\boxed{\text{Our Cavity}}$

Define sensitivity parameter: $S_1 \equiv \dfrac{\delta r_{c1}/w_a}{\delta\theta_1}$ as fraction of spot size per unit angle deviation

δr_{c1} = displacement from old axis inside the active medium (still modeled as a thin lens)

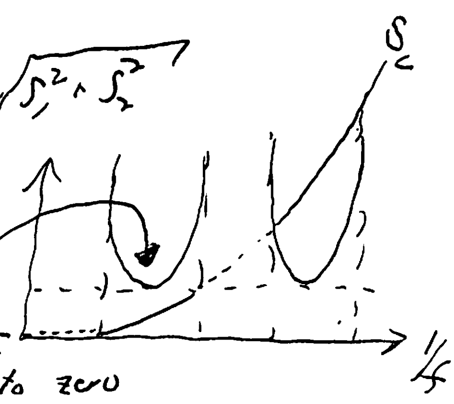
w_a = spot size in rod

$\delta\theta_1$ = tilt of mirror 1



Likewise $S_2 \equiv \dfrac{\delta r_{c2}/w_a}{\delta\theta_2}$

A combined figure of merit might be: $S_c \equiv \sqrt{S_1^2 + S_2^2}$



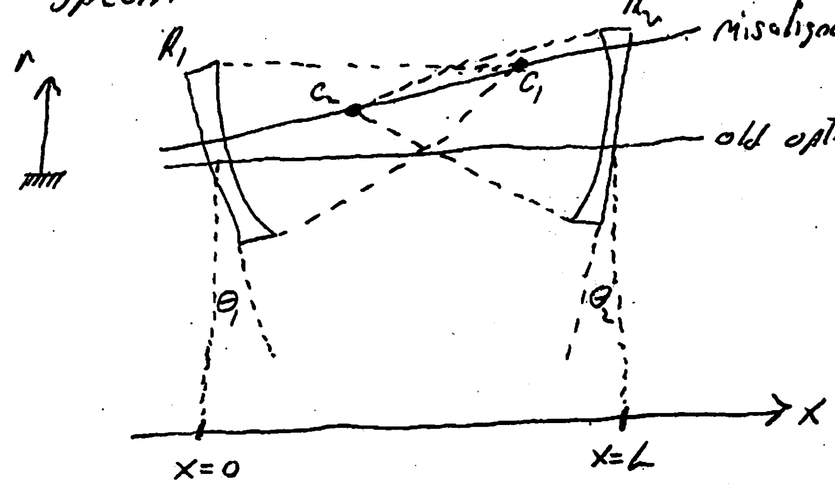
So you operate here

The weaker the lens the better.

not necessarily to zero

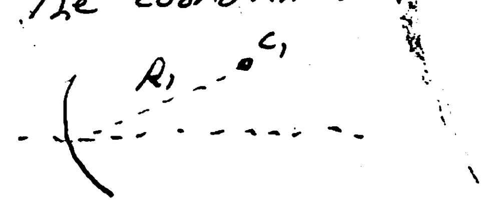
There are other ways to compensate for the thermal lens: addition of a lens or ...

I'm going to solve the misalignment problem for special cases. As I described in class, we have: a general two-mirror cavity first, before considering



Replace x with z.

The new optic axis connects c_1 and c_2. The coordinates of these points are:



$$c_1 \Rightarrow \quad x = R_1 \cos\theta_1 \approx R_1$$
$$r = R_1 \sin\theta_1 \approx R_1 \theta_1$$

$$c_2 \Rightarrow \quad x = L - R_2 \cos\theta_2 \approx L - R_2$$
$$r = R_2 \sin\theta_2 \approx R_2 \theta_2$$

Let the line connecting them be $r = mx + b$.

$$m = \frac{\Delta r}{\Delta z} = \frac{R_2\theta_2 - R_1\theta_1}{L - R_2 - R_1} \qquad b = R_1\theta_1 - mR_1$$

The position of the misaligned optic axis at mirror #1 is:

$$r_1 = b = \left[R_1 + \frac{R_1^2}{L - R_2 - R_1}\right]\theta_1 - \frac{R_1 R_2}{L - R_2 - R_1}\theta_2$$

$$r_1 = \frac{R_1 L - R_1 R_2}{L - R_1 - R_2}\theta_1 - \frac{R_1 R_2}{L - R_1 - R_2}\theta_2 \; \checkmark$$

We can write this using $g_1 \equiv 1 - \frac{L}{R_1}$ and $g_2 \equiv 1 - \frac{L}{R_2}$ by multiplying by $\frac{1/R_1R_2}{1/R_1R_2}$:

$$r_1 = \frac{\overbrace{\frac{L}{R_2} - 1}^{-g_2}}{\frac{L}{R_1R_2} - \frac{1}{R_2} - \frac{1}{R_1}}\,\theta_1 \overset{-}{\cancel{+}} \frac{1}{\frac{L}{R_1R_2} - \frac{1}{R_2} - \frac{1}{R_1}}\,\theta_2$$

Notice $g_1 g_2 = 1 - \frac{L}{R_1} - \frac{L}{R_2} + \frac{L^2}{R_1R_2}$, we get: $\frac{L}{R_1R_2} - \frac{1}{R_1} - \frac{1}{R_2} = \frac{-(1-g_1g_2)}{L}$

$$r_1 = \frac{g_2}{1-g_1g_2}\,L\theta_1 + \frac{1}{1-g_1g_2}\,L\theta_2$$

We get r_2 by swapping indices:

$$r_2 = \frac{1}{1-g_1g_2}\,L\theta_1 + \frac{g_1}{1-g_1g_2}\,L\theta_2$$

The angle of the misaligned axis wrt the old axis is:

$$\theta = \frac{r_2 - r_1}{L} = \frac{1-g_2}{1-g_1g_2}\,\theta_1 + \frac{g_1-1}{1-g_1g_2}\,\theta_2$$