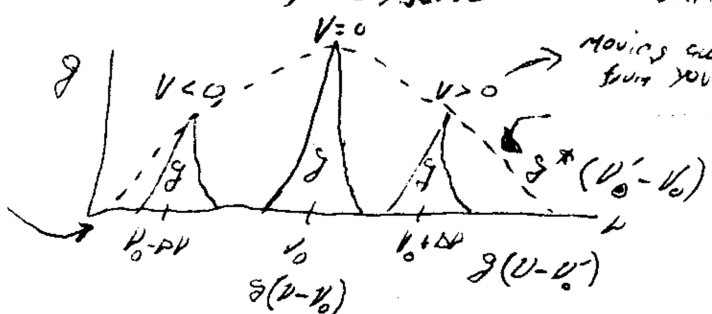
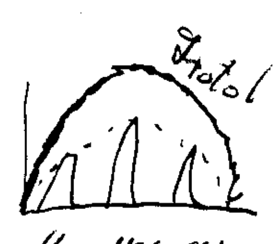
Now for the inhomogeneous case.

We imagine the total population can be divided into sub-populations, each w/ a homogeneous line shape.

Eg: Atomic gas. The atoms are in motion and their emitted radiation is Doppler shifted and the radiation they "observe" is shifted as well.

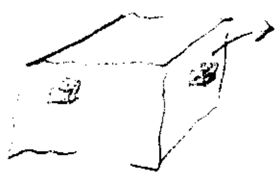
First draw all g's w/ same height $\int g\,d\nu = 1$





$\nu_0 = V = 0$ resonance
$\nu_0' =$ actual resonance

Eg: Solid state media



→ Different atoms, defects, etc.

$\int dN_t = N_t$

\# atoms/volume with resonance frequency between ν_0' & $\nu_0' + d\nu_0'$ (but w/ a standard line shape about ν_0')

$$\equiv dN_t \equiv N_t\, g^*(\nu_0' - \nu_0)\, d\nu_0'$$

We had $\dfrac{dP}{d\nu} = N_t\, W_h(\nu - \nu_0)\, h\nu$ ← Don't care; freq. of the light photon — whatever color we want it to be

absorb at ν_0'; resonance sub-pop dN_t

Power absorbed just by the dN_t per unit volume $=$

$$d\left(\frac{dP}{d\nu}\right) = dN_t\; W_h(\nu - \nu_0')\, h\nu$$

$$= \left[N_t\, g^*(\nu_0' - \nu_0)\, d\nu_0'\right] W_h(\nu - \nu_0')\, h\nu$$

$$\frac{dP}{d\nu} = N_t\, h\nu \int \underbrace{W_h(\nu - \nu_0')}\, \underbrace{g^*(\nu_0' - \nu_0)\, d\nu_0'}$$

↳ $\propto$ \# atoms/volume w/ resonance freq ν_0'

↳ response of a population of atoms w/ resonance freq ν_0'

For the homogeneous case we had.

$$\frac{dP}{dV} = N_t h\omega W_h$$

So we write for the inhomogeneous case:

$$\frac{dP}{dV} = N_t h\nu W_{in}$$

w/ $$\boxed{W_{in} = \int W_h(\nu - \nu_0') g^*(\nu_0' - \nu_0)\, d\nu_0'}$$

Smearing or broadening of homogeneous line

If we define:

$$\sigma_{in} = \frac{W_{in}}{F}$$

we get

$$\boxed{\sigma_{in} = \int \sigma_h(\nu - \nu_0') g^*(\nu_0' - \nu_0)\, d\nu_0'}$$

This is an <u>average cross-section</u> that we can use to characterize the whole system! (For some problems)

Note : $\sigma_h = \sigma_h(\nu) \qquad \propto \nu g(\omega - \nu_0)$

$\sigma_{in} = \sigma_{in}(\omega) \qquad \propto \int \cdots d\nu_0'$

Recall, we had:

$$\sigma_h = \frac{2\pi^2}{3n\varepsilon_0 c h} |\mu|^2 \nu \, \underbrace{g(\nu-\nu_0)}_{\text{atomic line shape}}$$

We can write:

$$\sigma_{in} = \frac{2\pi^2}{3n\varepsilon_0 c h} |\mu|^2 \nu \, \underbrace{g_t(\nu-\nu_0)}_{\text{total line shape}}$$

$\sigma_{in} = \frac{W_{in}}{F} \ldots$

$$g_t = \int_{-\infty}^{\infty} g^*(x)\, g(\nu-\nu_0-x)\, dx$$

x = change in resonance freq.

$x = \nu_0' - \nu_0$

limiting case #1) No inhomogeneous broadening

$$g^* = \delta(x)$$

$$g_t = g(\nu-\nu_0)$$

#2) Narrow atomic line

$$g = \delta(\nu-\nu_0-x)$$

$$g_t = g^*(\nu-\nu_0)$$

e.g. Doppler shift provides all the broadening

Following the text, from now on, we'll just use:

$$\boxed{\begin{gathered}\sigma = \frac{2\pi^2}{3n\varepsilon_0 c h} |\mu|^2 \nu \, g_t(\nu-\nu_0) \\ W = \sigma F\end{gathered}}$$

and not specify homo. or inhomo. broadening explicitly. That's all in $g_t()$.

So, how does general 2-level system affect the light?

We had $\Rightarrow$ $dF_{abs} = -\sigma N_t F\,dz$ All atoms in lower state, N_t = # density

$N_t \equiv N_1$

$dF_{stim\,em} = +\sigma N_t F\,dz$ All atoms in upper state

$N_t \equiv N_2$

$\boxed{\text{General}}$ N_1 in lower state, N_2 in upper

$$dF = -\sigma(\underbrace{N_1}_{abs} - \underbrace{N_2}_{stim\ em.})F\,dz$$

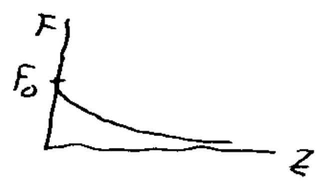
$dF = dF_{abs} + dF_{stim\,em}$

$\boxed{\text{Assume } N_1 > N_2}$

$\alpha \equiv \sigma(N_1 - N_2)$ = absorption coefficient ≥ 0

$$\frac{dF}{dz} = -\alpha F$$

$$F(z) = F_0 e^{-\alpha z}$$



$$\alpha = \frac{1}{\text{length}}$$

$\alpha = 2\,cm$ (constant sleeps used) means flux falls by 1/e in 2 cm

$\boxed{\text{But } N_2 > N_1 \text{ (population inversion)}}$

$$\gamma \equiv \sigma(N_2 - N_1) \geq 0$$

$$\frac{dF}{dz} = \gamma F$$

$$F(z) = F_0 e^{\gamma z}$$

Of course, once F becomes large enough, its absorption/amplification affects $(N_1 - N_2)$

It drives it to zero. No absorption or gain.

If $F = 0$ $F(z) = 0$.

Einstein thermodynamic treatment (Non-degenerate treatment)

How are stim em, spon em & abs related?

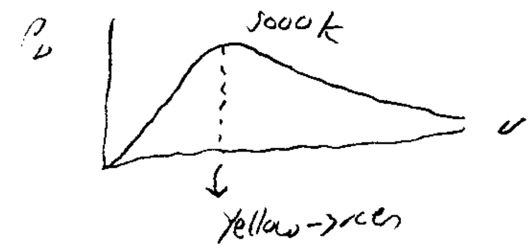
Even w/o a quantum treatment of light, we can use nature and a little physics to say some things about spon emission.

Suppose we have a blackbody at temp T.
By def'n, it is at equilibrium.
(We'll wait until it is.)

We know (Eq 2.2.22)

energy density
$$\rho(\nu) = \frac{8\pi \nu^2 n^3}{c^3}\,\frac{h\nu}{e^{h\nu/kT}-1}$$

$\boxed{\text{Don't ERASE}}$



Now, we have abs, stim em, spon em taking place in equilibrium

$W_{21}^{SA} \propto I \propto \rho$

Characterize the rates as:

$W_{21}^{SA} = B_{21}\,\rho_{\nu_0}$ stim em. $\qquad \rho_{\nu_0} = \rho(\nu_0)$

$W_{12}^{SA} = B_{12}\,\rho_{\nu_0}$ abs

$W_{21,\,spon}^{SA} = A$ spon-em

energy density is more conventional for a completely closed cavity. Light's not going anywhere.

B_{12}, B_{21}, A contain all atomic information now.

We require:

$$B_{21}\rho_{\nu_0}N_2 + AN_2 = B_{12}\rho_{\nu_0}N_1$$

ρ_{ν_0} & N_1 & N_2 independent of time.

However, from the Boltzmann distribution:

$$\frac{N_2}{N_1} = e^{-h\nu_0/kT}$$

(Non-degenerate case. Extension to degenerate case not interesting to us.)

$$B_{21}\,\rho_{\nu_0}\,\cancel{N_1} + A\,\cancel{N_2} = \frac{B_{12}\,\rho_{\nu_0}\,\cancel{N_2}}{e^{-h\nu_0/kT}}$$

$$\rho_{\nu_0} = \frac{A}{B_{12}e^{h\nu_0/kT} - B_{21}} = \frac{A/B_{21}}{\frac{B_{12}}{B_{21}}e^{h\nu_0/kT} - 1}$$

Compare to $\rho(\omega)$ above:

$$\frac{B_{12}}{B_{21}} = 1 \quad \Rightarrow \quad B_{12} = B_{21} = B \quad \checkmark$$

$$\frac{A}{B} = \frac{8\pi h \nu_0^3 n^3}{c^3}$$

Text shows,

$$B \propto |\mu|^2$$

so $A \propto |\mu|^2$.

A strong transition will suffer large spon. emission losses.

$$\rho(\nu) = \frac{8\pi\nu^2 n^3}{c^3}\,\frac{h\nu}{e^{h\nu/kT}-1}$$

$$\rho = \frac{A/B_{21}}{\frac{B_{12}}{B_{21}}e^{h\nu/kT}-1}$$

$$\frac{B_{12}}{B_{21}} = 1 \qquad B_{12} = B_{21} \equiv B$$

$$\frac{A}{B} = \frac{8\pi h \nu_0^3 n^3}{c^3}$$

Text then goes on to show:

$$A = \frac{16\pi^3 \nu_0^3 n |\mu|^2}{3 h \varepsilon_0 c^3}$$

$$B = \frac{2\pi^2 |\mu|^2}{3 n^2 \varepsilon_0 h^2}$$

→ We have basic interaction of EM wave & atom

W_{abs}, W_{stim} by calculation ⇒ τ $\sigma \to \mu, g$

A by formula ⇒ $\sigma \to \mu, g$

Next → look at forms of g

Then → remove some of our approximations: 2-level system