Coherence (Chap 11)

Text argues the most characteristic properties of a laser are:

monochromaticity coherence (spatial & temporal)

directionality brightness

These properties are inter-related and tied physically to the heart of a laser: gain via stimulated emission.

To get started, let's assume a quasi-monochromatic wave:

$$E(\vec{r},t) = A(\vec{r},t)\, e^{i(\bar{\omega}t - \phi(\vec{r},t))}$$

$\bar{\omega}$ = average ω

A & ϕ are slowly varying: $\frac{1}{A}\frac{\partial A}{\partial t}$ & $\frac{\partial \phi}{\partial t} \ll \bar{\omega}$

$$I(\vec{r},t) = EE^* = |A(\vec{r},t)|^2$$

Let's also assume a "stationary" wave:

statistical and average measures of the field (like I) vary slowly with time

The simplist example of a non-stationary field is that from a Q-switched pulsed or modelocked pulsed laser.

A stable thermal source is stationary, but not usually described by the formula above.

What phenomena should we look at to best understand light?

As a first guess, we should choose phenomena that are field-dependent and not simply intensity dependent. Interference effects are an obvious possibility. However, the final measurement is always an intensity (or something like one).

If we combine two fields, $E(t) = E_1(t) + E_2(t)$ the intensity is proportional to:

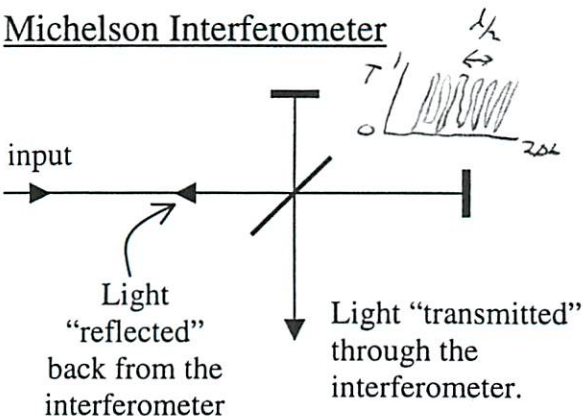
$$I \sim \langle E(t)\rangle^2 = \langle E_1\rangle^2 + \langle E_2\rangle^2 + 2\langle E_1E_2\rangle$$

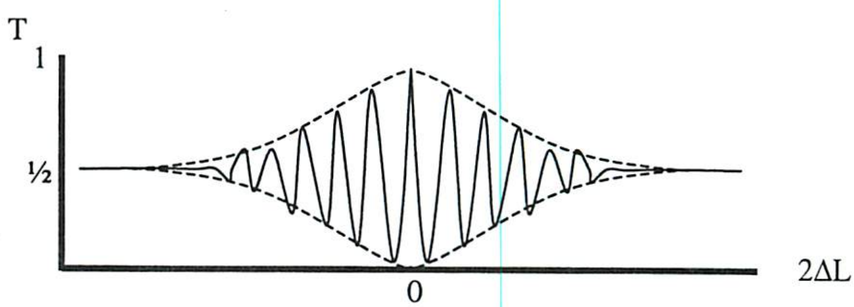
where <> denotes time average over an appropriate time (one period for quasi-monochromatic light). If E_1 and E_2 are completely uncorrelated, the time average will yield zero for the cross-term and:

$$I = I_1 + I_2 = 2 I_0$$

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} = 4 I_0$$

Michelson Interferometer





If the input field is a true sinusoid, looking at the "transmitted" light we would have something like:

$$E_1 = E_o \cos\omega t \;\&\; E_2 = E_o \cos(\omega t + \phi)$$

in which case the output intensity is:

$$I = \tfrac{1}{2} I_o + \tfrac{1}{2} I_o \cos\phi$$

where I_o is the input intensity. In reality, the input field will have phase and amplitude fluctuations, resulting in a transmission pattern like the graph above.

The "coherence length" L_c is, loosely defined, the distance $2\Delta L$ by which the two arms can differ and still yield interference. The "coherence time" is $t_c = L_c/c$. A pure sinusoid would have infinite coherence time.

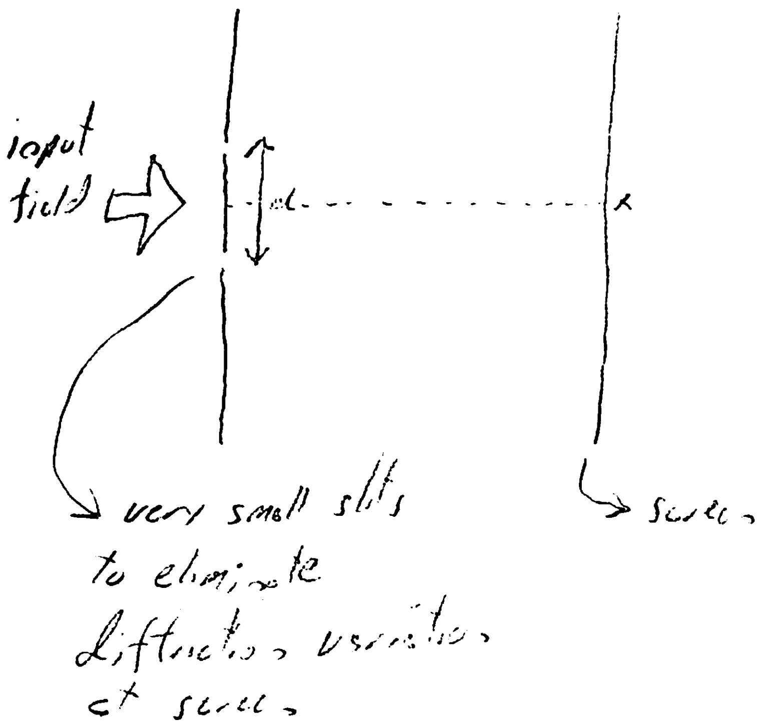
Caveat: "Coherence" is defined by and measured via correlations. There are many measures one can imagine and $\langle E_1E_2\rangle$ is only the simplest. Accordingly, this lecture is restricted to the simplest kind coherence, called first-order.

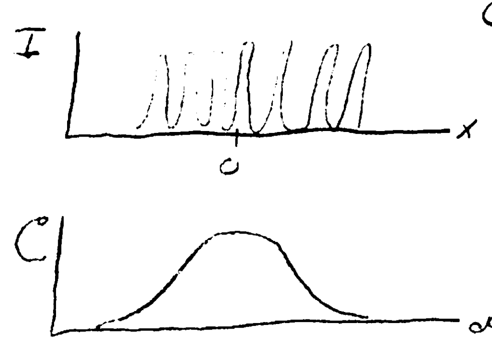
If the bandwidth of the light is $\Delta\nu$, a good estimate for quasi-monochromatic, stationary light is:

$$t_c = 1/\Delta\nu.$$

{Clearly, for any of this to make sense, we need a formal, consistent procedure for defining t_c and $\Delta\nu$. As we broaden our study, we'll also need to define spot sizes, divergence angles and more. This isn't just a factor of 2 or π. What's the spectral width of an arbitrarily messy spectrum? Consistent choices exist, but since this isn't a quantitative treatment, let's defer that discussion.}

We can perform a spatial equivalent interference experiment using Young's Apparatus:





$$\text{Contrast} \equiv \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

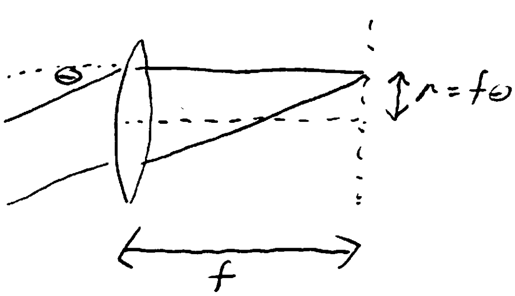
This apparatus can be used to define an area over which the light is coherent.

The spatial equivalent to the bandwidth turns out to be the divergence angle of the beam in the far field or, more generally, the directionality.

We've considered two ways to measure directionality so far:

① $\Theta = \frac{W(z)}{z}$ ⇨ half-angle divergence — depends on definition of the spot-size $W(z)$

② Measure the "angular content" using a lens.



$$\begin{pmatrix} r \\ r' \end{pmatrix} = \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}\begin{pmatrix} r_{in} \\ r_{in}' \equiv \theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix}\begin{pmatrix} r_{in} \\ -r_{in}/f + \theta \end{pmatrix}$$

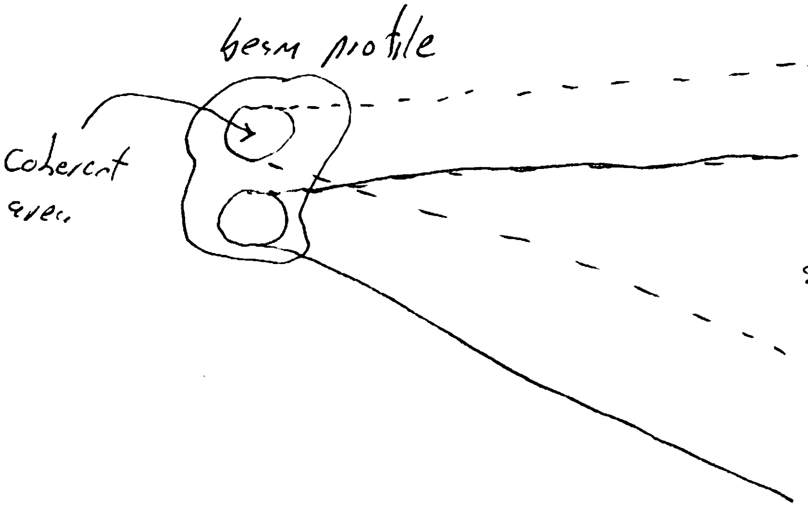
⇩

$$r = r_{in} - f\left(\frac{r_{in}}{f}\right) + f\theta$$

$I(r) \rightarrow I(\theta)$

Methods #1 & #2 are related, in fact.

In general, for two beams with the same intensity profile $I(x,y)$, the one with the best spatial coherence will diverge less.



Before we used: $\Theta = B\lambda/D$

$$\Downarrow$$

$$\Theta = B\frac{\lambda}{D_c}$$

w/ D_c = diameter of coherent area

A TEM_{00} has the smallest B.
(Statement assumes a consistent method to measure Θ exists.)

M^2 Parameter ("beam quality")

$M^2 \geq 1$ and $M^2_{Gaussian} = 1$ (If not cylindrically symmetric, you have M^2_x and M^2_y)

$$\boxed{\Theta = M^2 \frac{\lambda}{\pi W_0}}$$ (for field) W_0 = spot size @ $z=0$.

Beam position: $\langle x \rangle \equiv \dfrac{\int x\, I(x,y,z)\,dx\,dy}{\int I(x,y,z)\,dx\,dy}$

$\langle x \rangle$ = function of z
Likewise for $\langle y \rangle$

Beam standard deviation $\sigma_x^2 \equiv \dfrac{\int (x-\langle x \rangle)^2\, I\,dx\,dy}{\int I\,dx\,dy}$

$W_{x0} \equiv 2\sigma_{x0} \Rightarrow$ For a Gaussian $W = W(z)$.

A taste of the formalism

Ensemble average $\overset{\text{first order}}{\Gamma^{(1)}}(\vec{r}_1, \vec{r}_2, t_1, t_2) \equiv \langle E(\vec{r}_1, t_1)\, E^*(\vec{r}_2, t_2)\rangle$

$\langle\,\rangle$ = average over many measurements. $E()$ can be non-stationary.

Temporal coherence of stationary beams:

- ensemble average can be a time average
- t_1 and t_2 don't matter separately, only $\tau \equiv t_1 - t_2$
- both field terms evaluated at the same location

$$\Gamma^{(1)}(\vec{r}, \tau) = \langle E(\vec{r}, t+\tau)\, E^*(\vec{r}, t)\rangle$$
$$= \lim_{T\to\infty} \frac{1}{T}\int_0^T E(\vec{r}, t+\tau)\, E^*(\vec{r}, t)\, dt$$

Complex coherence $\equiv \gamma^{(1)}(\vec{r}, \tau) = \dfrac{\Gamma^{(1)}}{\langle E(\vec{r}, t) E^*(\vec{r}, t)\rangle^{1/2} \langle E(\vec{r}, t+\tau) E^*(\vec{r}, t+\tau)\rangle^{1/2}}$

degree of temporal coherence $\equiv |\gamma^{(1)}(\vec{r}, \tau)| \leq 1 \Rightarrow$ 0 = incoherent, 1 = perfect coherence

$|\gamma^{(1)}(\vec{r}, 0)| = 1$ The coherence time τ_c is defined: $|\gamma^{(1)}(\vec{r}, \tau_c)| = \frac{1}{2}$

Spatial coherence:

$$\gamma^{(1)}(\vec{r}_1, \vec{r}_2, t) \equiv \frac{\langle E(\vec{r}_1, t)\, E^*(\vec{r}_2, t)\rangle}{\langle E(\vec{r}_1, t) E^*(\vec{r}_1, t)\rangle^{1/2} \langle E(\vec{r}_2, t) E^*(\vec{r}_2, t)\rangle^{1/2}}$$

degree of spatial coherence $\equiv |\gamma^{(1)}(\vec{r}_1, \vec{r}_2, t)|$ coherence area near $\vec{r}_1$ = area for which $|\gamma^{(1)}(\vec{r}_1, \vec{r}_2, t)| \geq 1/2$

... $|\gamma^{(1)}(\vec{r}, \vec{r}, ...)|$ at the screen center