Quick and dirty integration:

Consider the equation $dN_2/dt = -1/\tau\ N_2$. Let $N_2(0) = 10^{10}\ cm^{-3}$ and $\tau = 2.0\ \mu s$. Although we know the solution is $N_2(t) = N_2(0)\ e^{-t/\tau}$, let's try solving it numerically. We can represent the solution using two arrays $N_2(i)$ and $t(i)$ for the population and the time where i is an integer with $i = 1, 2, \ldots, i_{max}$. In other words, we are only going to try to determine N_2 at successive discrete values of time so, for example, at time t(6) the population is $N_2(6)$. So long as the time intervals are small compared to the characteristic time or times of the problem, this should be okay.

Let's guess that $\Delta t \equiv t(i+1) - t(i) = 0.05\ \tau$ is a good starting point for the time between evaluations and that we need to integrate out to $t_{max} = 4\tau$. One algorithm that might work is the following:

Initialize: $N_2(1) = 10^{10}\ cm^{-3}$ and $t(1) = 0$ s, i = 1.

→ Evaluate the rate at time t(i): $R \equiv dN_2/dt = -1/\tau\ N_2(i)$
Evaluate the population at $t(i+1) = t(i) + \Delta t$: $N_2(i+1) = N_2(i) + R\Delta t$.
$i = i+1$
Repeat until $i = i_{max} = t_{max}/\Delta t$

After trying your first integration (by whatever method you happen to choose) you should generally try a different value of Δt. If the approximation is to be valid, the result should be independent of Δt over some reasonable range. Also, you can usually tell by inspection if your initial choice for t_{max} is correct and adjust accordingly.

For simple equations where not much accuracy is required, this will actually work.

In general, this approach is a disaster because the error typically scales with Δt. There are methods whose error scales as $(\Delta t)^4$. In addition, the method given above is not robust with respect to numerical error that arises from the way computers represent numbers. For the needs of this course and the equations we are going to encounter, the results using this "quick and dirty" approach will be acceptable. I strongly encourage you to find an approach that will assist you in your research as the years go by. Having to integrate equations of motion is a common task.