Skipping Fourier Analysis: 5.4, 5.5
and wavepackets 5.6.

Group and Phase Velocity (5.7)

Harmonic wave: 

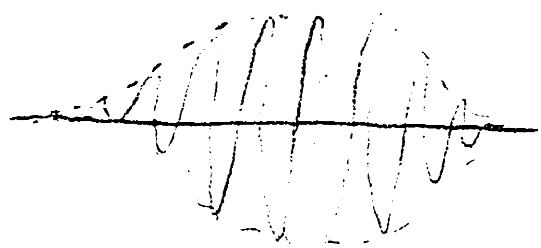
$\phi = kx - \omega t$ = constant identifies a point on the wave

$$\frac{d\phi}{dt} = 0 \qquad k\frac{dx}{dt} - \omega = 0 \qquad \frac{dx}{dt} = \frac{\omega}{k} \qquad \boxed{V_p = \frac{\omega}{k}} \text{ phase velocity}$$

$$\boxed{V_p = \frac{c}{n} = f\lambda}$$

Not observable for reasons similar to plane waves not being observable.

Real observable wave, called a wavepacket, turns on and off.
Can be idealized as a pulse:



which is a superposition of harmonic waves.

Let $\Phi(k) = kx - \omega t$ give the phase of each freq component. Note $k = k\omega$.

At the peak: $\frac{d\Phi}{dk} = 0$

$$x - \frac{d\omega}{dk}t = 0$$

$$\frac{x}{t} = \frac{d\omega}{dk}$$

x at must be identified now as the position of the peak at time t

$\rightarrow$ $V_g = \frac{d\omega}{dk}$

This is observable and well defined so long as the wavepacket isn't changing shape too severely.

This is the speed of energy transport: $V_g < c$.

$\omega = \omega(k)$ is called a dispersion relation.

$\omega = v_p k = \frac{c}{n} k$ is the one we know best.

It's a different one for a waveguide.

$$V_p = \frac{\omega}{k} = \frac{c}{n} \checkmark$$

$$V_g = \frac{d\omega}{dk} = \frac{d}{dk} V_p k = V_p + k \frac{dV_p}{dk}$$

$$\frac{dV_p}{dk} = \frac{d}{dk} \frac{c}{n} = -\frac{c}{n^2} \frac{dn}{dk}$$

$$V_g = V_p - \frac{kc}{n^2} \frac{dn}{dk}$$

$$V_g = V_p \left(1 - \frac{k}{n} \frac{dn}{dk}\right) \checkmark$$

$$= V_p \left(1 + \frac{\lambda}{n} \left(\frac{dn}{d\lambda}\right)\right) \checkmark$$

Good for use with a Sellmeier eqn.

$$= \frac{c}{n + \omega \frac{dn}{d\omega}} \equiv c/n_g \checkmark$$

In analogy to $V_p = c/n$

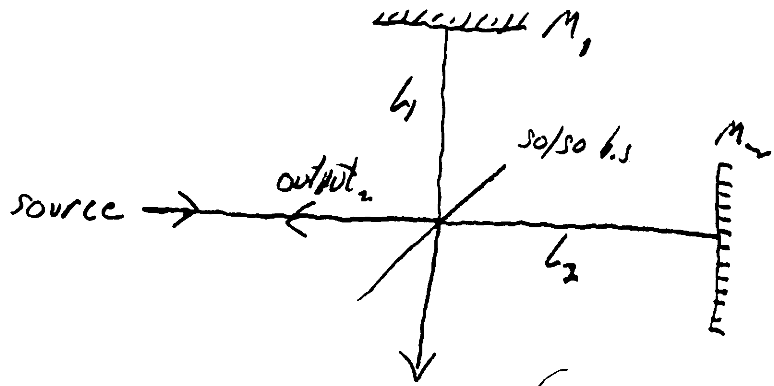
See Ex 5.10 for a treatment of conductors.

In normal media: $\frac{dn}{d\omega} > 0$ so $n_g > n$

and $V_g < V_p$ by about $\approx 1\%$.

Interferometry (S.8)

Michelson Interferometer (simple) in air



$$\left\{ \begin{array}{c} \text{refl off b.s} \rightarrow M1 \rightarrow \text{trans through b.s} \\ + \\ \text{trans through b.s} \rightarrow M2 \rightarrow \text{refl off b.s} \end{array} \right\}$$

Assuming flat phase fronts and ignoring phase shifts from reflections and transmissions:

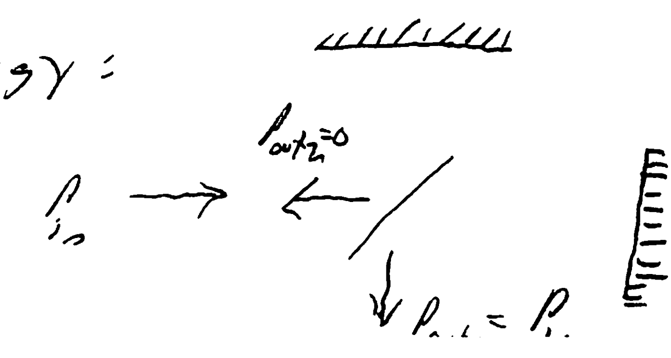
$$\Delta\delta = \frac{2\pi\,(\text{path length difference in air})}{\lambda}$$

$$= 2\pi\,\frac{2L_2 - 2L_1}{\lambda} = 4\pi\,\frac{\Delta L}{\lambda} \checkmark$$

$\Delta\delta = N 2\pi$ for fully constructive interference

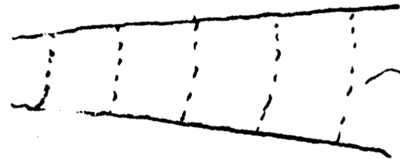
$\Delta L = N \lambda/2$ ✓

Conservation of energy:



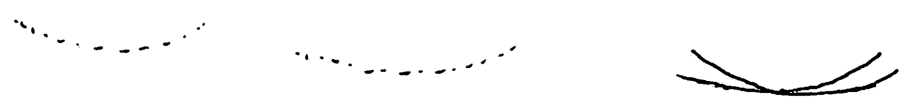
In general, the source is converging or diverging.

Model this as:



For output, at $\Delta L = 0$

But for $L_2 > L_1$



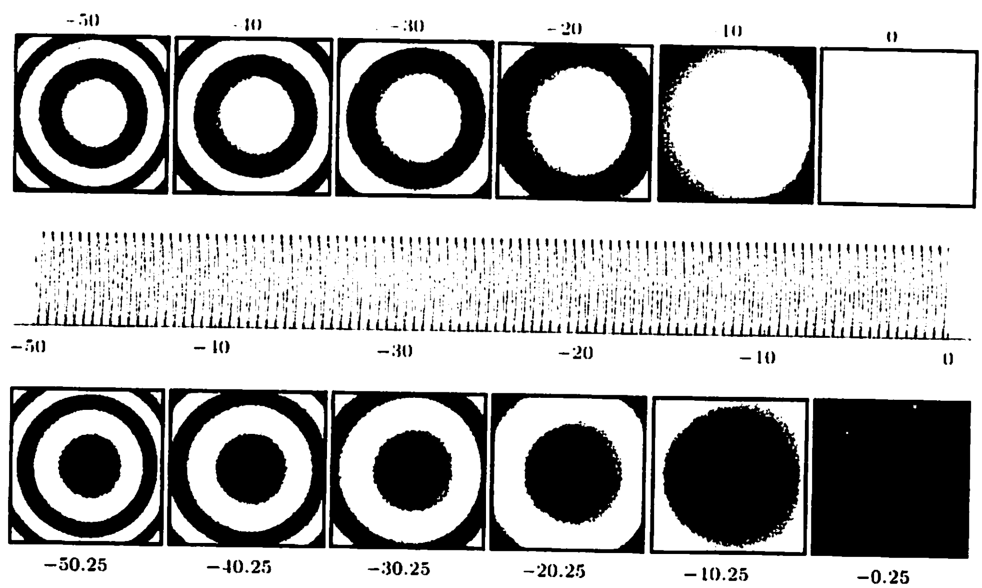
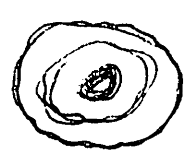


Figure 5.23 Fringe patterns as ΔL approaches zero. The patterns below the interferogram are the complements of those above. At $\Delta L = 0$, the entire field of one interferometer output is bright, and the other field is dark.

If at $L = 0$ you see  one of your components isn't flat. This is a good way to measure surfaces!

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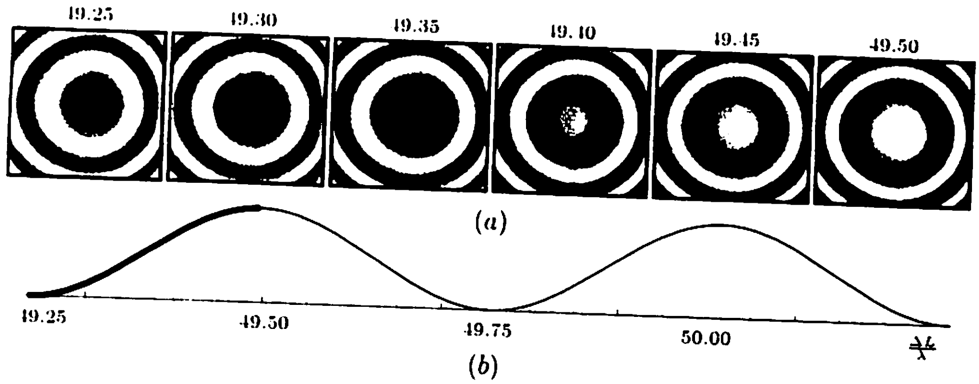


Figure 5.21 (a) Fringe variation in a Michelson interferometer as the mirror is moved by $\lambda/4$. (b) Interferogram. The thick segment corresponds to the fringes in (a).

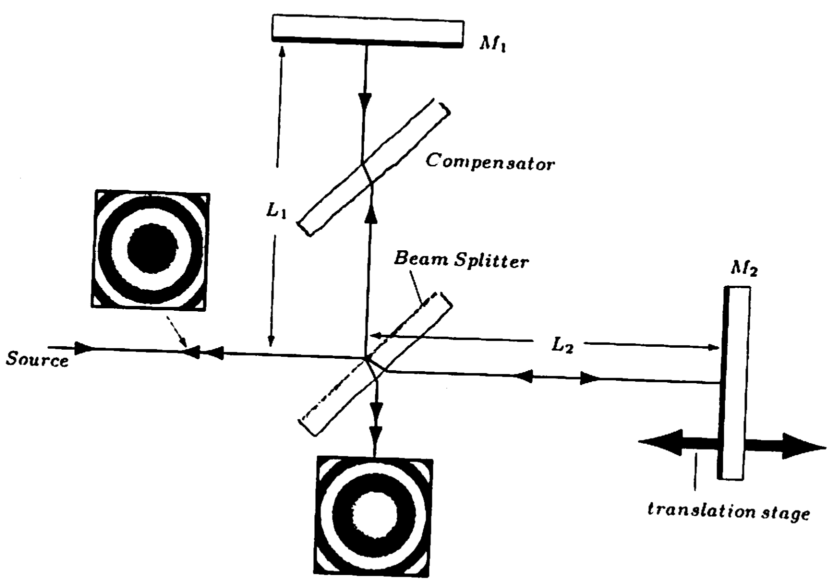


Figure 5.20 A Michelson interf

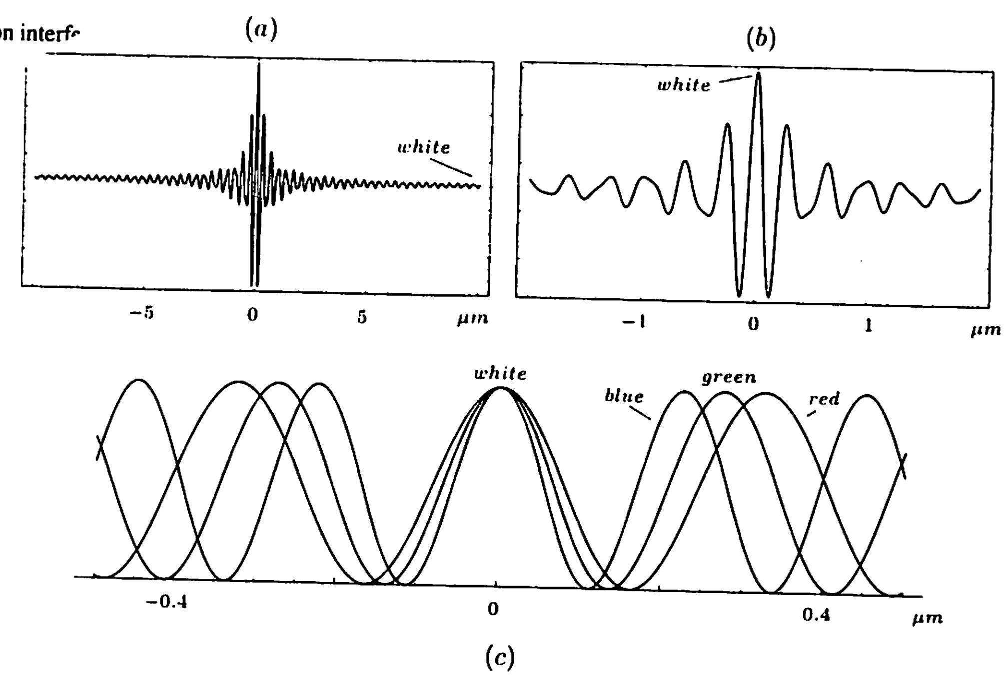


Figure 5.24 (a) Interferogram for white light. (b) Zoom detail of (a). (c) Interferograms for different colors combine to give a white fringe at $\Delta L = 0$, and again for larger values of ΔL

Multiple-Beam Interference (5.10)

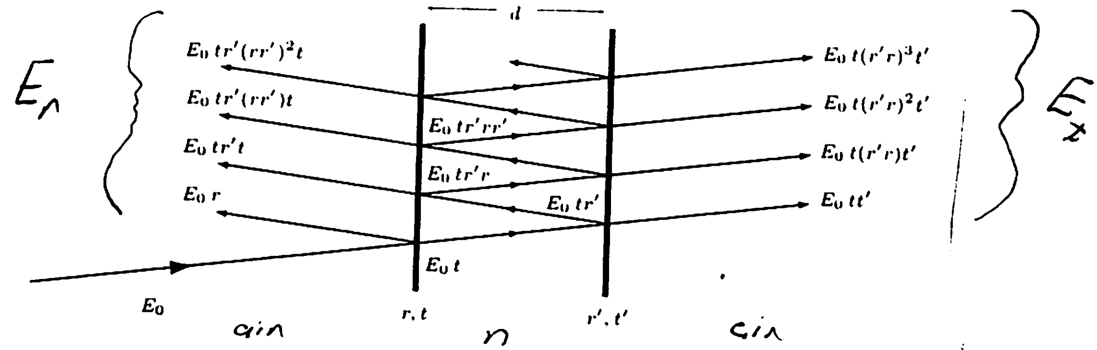


Figure 5.27 Multiple beam interference between two parallel partially reflecting surfaces

This configuration is often used at normal incidence so we'll solve for that case to simplify things a little.

Usually $r = r'$ & $t = t'$ so we'll put that in shortly

Round trip phase difference $\equiv \delta_0 = 2 \times 2\pi \dfrac{nd}{\lambda_0} = k_0(2nd)$

$\quad$ ⟶ text uses k here

We can ignore the vector character of the wave.

$$E_t = E_0 t e^{i\delta_0/2} t' + E_0 t e^{i\delta_0/2} r r' e^{i\delta_0} t'$$

$$+ E_0 t e^{i\delta_0/2} (r r' e^{i\delta_0})^2 t' + \dots$$

$$= E_0 t t' e^{i\delta_0/2} \left[1 + r r' e^{i\delta_0} + (r r' e^{i2\delta_0})^2 + \dots\right]$$

⟶ By convention, we usually drop this term.

The text never mentions it.

Now $|rr' e^{i\delta_0}| = |rr'| < 1$ so use

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \ldots = \frac{1}{1-x}$$

$$E_t = E_0 t t' \frac{1}{1 - rr' e^{i\delta_0}} = E_0 \frac{t^2}{1 - r^2 e^{i\delta_0}}$$ for $r = r'$, $t = t'$

$$E_t^* = E_0^* \frac{t^{*2}}{1 - r^{*2} e^{-i\delta_0}}$$

$$I_t = I_0 \frac{(tt^*)^2}{1 - (r^2 e^{i\delta_0} + r^{*2} e^{-i\delta_0}) + (rr^*)^2}$$

At normal incidence $tt^* = T$ + $rr^* = R$

Let $r = r_0 e^{i\phi_R}$ where ϕ_R = reflection phase shift

$r^2 = r_0^2 e^{i2\phi_R}$ For an uncoated glass slab $\phi_R = 0$

$= R e^{i2\phi_R}$ However usually multilayer coated slabs are used, so we leave ϕ_R in.

$$I_t = I_0 \frac{T}{1 - (R e^{i\delta} + R e^{-i\delta}) + R^2}$$ where $\delta \equiv \delta_0 + 2\phi_R$

$$= \frac{4\pi n d}{\lambda_0} + 2\phi_R$$

$$= I_0 \frac{T}{1 - 2R\cos\delta + R^2}$$

This is fine, but there's a standard form that's always used so work on this a little more:

$$1 - 2R\cos\delta + R^2 = (1-R)^2 + 2R(1-\cos\delta)$$

$$= (1-R)^2 + 2R\left(2\sin^2\frac{\delta}{2}\right)$$

$$= (1-R)^2\left[1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}\right]$$

Define the Finesse coefficient $F \equiv \dfrac{4R}{(1-R)^2}$

$$I_t = I_0 \frac{T}{(1-R)^2} \frac{1}{1+F\sin^2\frac{\delta}{2}} \quad \checkmark$$

For no absorption: $T + R = 1$ (This is not always the case. Absorbing media are sometimes deliberately introduced for spectroscopic analysis.)

$$I_t = I_0 \frac{1}{1+F\sin^2\delta/2} \quad \checkmark$$

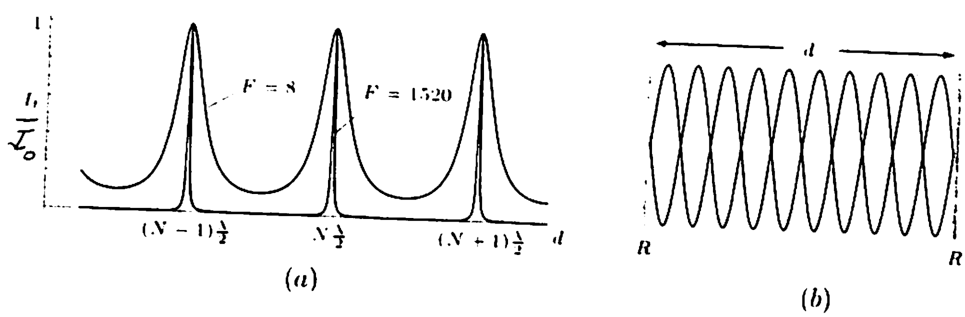


Figure 5.28 (a) Transmittance of two identical flat parallel mirrors separated by distance d. A mirror reflectance of 0.5 gives $F = 8$, and $R = 0.95$ gives $F = 1520$. (b)Standing waves when the mirror spacing d is equal to an integer number half wavelengths.

Or think of graph (a) as I_T/I_0 vs f → spectroscopy!

- Even if $R = 0.95$, $I_T/I_0 = 1$ for some f !!

$I_T = I_0$ when $\sin^2 \frac{\delta}{2} = 0$ or

$$\frac{2\pi n d_N}{\lambda_0} + \phi_R = N\pi \qquad N \in \mathbb{Z}$$

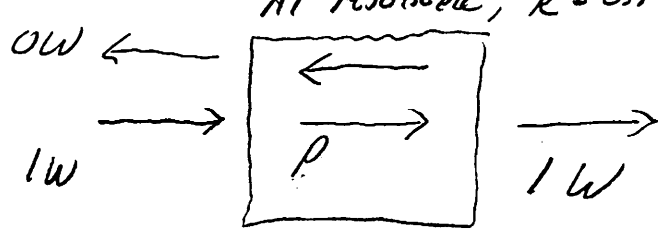
$$d_N = \frac{\lambda_0}{2\pi n}\left(N\pi - \phi_R\right)$$

$$d_{N+1} - d_N = \frac{\lambda_0}{2n} = \frac{1}{2}\lambda \checkmark$$

This is a resonance effect: resonant cavity

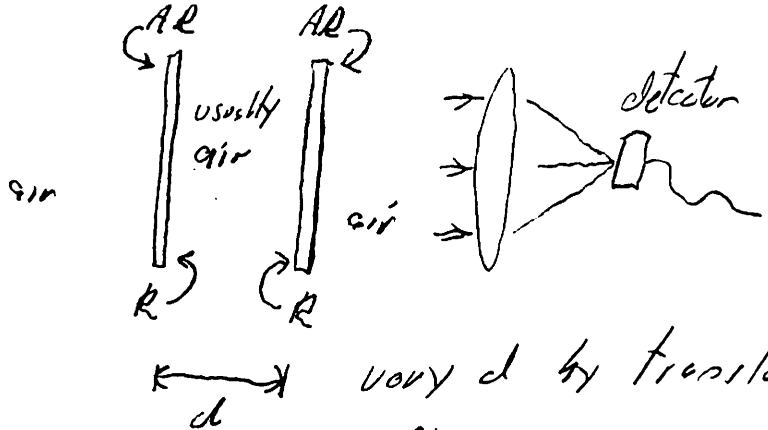
Note that a field analysis shows a consistent result:

At resonance, $R = 0.95$



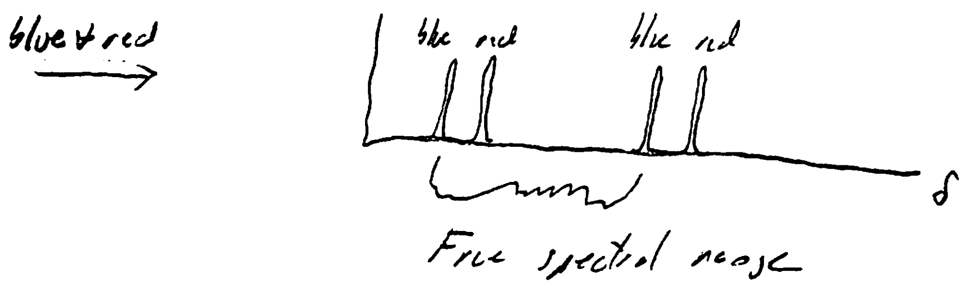
$$P(1-R) = 1W \qquad P = \frac{1W}{0.05} = 20W$$

Scanning Fabry-Perot Interferometer



The mirrors must be very flat and very parallel.

vary d by translating one of the mirrors, often using a piezoelectric transducer.



$$\frac{I_t}{I_0} = \frac{1}{1 + F\sin^2\frac{\delta}{2}} = \frac{1}{2}$$

$$\sin\frac{\delta_{1/2}}{2} = \frac{1}{\sqrt{F}}$$

If we're doing spectroscopy, F is large so $\delta_{1/2}$ is small

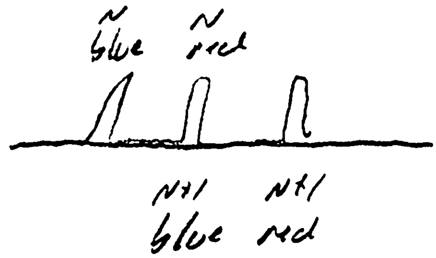
$$\delta_{1/2} = \frac{2}{\sqrt{F}}$$

$$FWHM = \frac{4}{\sqrt{F}}$$

Finesse $\mathcal{F} \equiv \frac{2\pi}{FWHM} = \frac{\pi\sqrt{F}}{2}$

Larger Finesse yields better resolution.

It can happen that the spectrum to be measured is too broad:



$\nearrow$ assume small in comparison

$$\frac{2\pi n d}{\lambda_0} + \phi_R = N\pi$$

$$N = \frac{2nd}{\lambda_0} = \frac{2d}{\lambda}$$

For overlap to just occur:

$$(N+1)\lambda = N(\lambda + \Delta\lambda) \qquad \Delta\lambda = \lambda_{red} - \lambda_{blue}$$

$$\Delta\lambda = \frac{\lambda}{N} \checkmark$$

text has $\frac{\lambda^2}{2nd}$

Free spectral range $\Delta\lambda_{FSR} = \frac{\lambda^2}{2d}$ ✓

For λ, use $\bar{\lambda}$ of the spectrum to be analyzed.

Keep spectral width well below λ_{FSR}.

In frequency: $\boxed{\Delta f_{FSR} = \frac{c}{2nd}}$ ✓