METRIC PREFIXES AND VECTOR REVIEW

Metric prefixes:

From Coulomb's Law you can see that 1 Coulomb is an enormous amount of charge. You will also learn that 1 Farad is an enormous capacitance and 1 Tesla is a large magnetic field, whereas1 eV is a minuscule amount of energy. We will need to use metric prefixes so we can talk about these things comfortably. Here is a review:

G (giga)	M (mega)	k (kilo)	c (centi)	m (milli)	μ (micro)	n (nano)	p (pico)
10^9	10^6	10^3	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}

So, now we can say that the charge transferred in the static electricity demos was about 1 nC. My exams are written on the assumption that you know these!

Vectors:

Now, the electric force, like all the other forces you have seen, is a vector quantity. We denote vectors by writing them in boldface, or with an arrow on top, for example, **F** or $\vec{F}$. If you see a vector quantity written with a normal type face, as in "a force F of 6 N," then just the magnitude is being referred to. A vector is a quantity that has both a direction and a magnitude. If we ask for a vector, and you only give a magnitude, you haven't completed the problem. Vectors are generally three dimensional, but most of the problems we will consider at first are two dimensional, so we will limit discussion to that case for now.

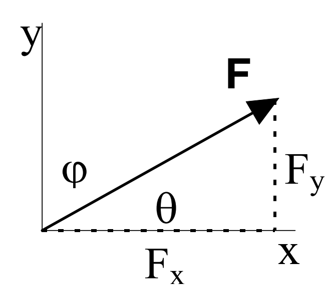
Here are three ways to specify a vector:

- Magnitude and direction. For example, "the electric force is 6 N in the +x direction" or "the velocity is 2 m/s northward." Note that the first statement means *nothing* unless there is an accompanying figure with coordinate axes on it showing how the x-axis is oriented.
- Components. For example, "the electric force is **F** = 2**i** + **j**," where **i** and **j** are the unit vectors pointing in the +x and +y directions. Also good is "F_x = 2 N and F_y = 1 N," were F_x means the x-component of F, etc. In either case, however, this also means *nothing* without a figure.
- A description. For example, "The force on charge Q_1 is 3 N, directed towards charge Q_2."

Going between magnitude/direction and components descriptions.

Take the above example, **F** = 2**i** + **j**.

Magnitude: $F = \sqrt{F_x^2 + F_y^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$.



Since we are in two dimensions (that is, the vector is given to lie in the xy plane) we only need give the angle, θ, with respect to the x-axis to specify a direction.

Direction: $\tan\theta = F_y / F_x = 1/2$, so $\theta = \tan^{-1}(0.5) = 26.6$ degrees.

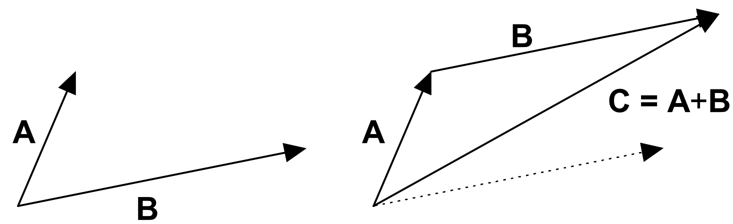
You could also give a different angle, say φ. Just be clear which angle you are using. Finally, if you have magnitude and direction, then you can get the components.

Components: $F_x = F \cos\theta$

$F_y = F \sin\theta$.

Adding vectors. There are two ways to add vectors:

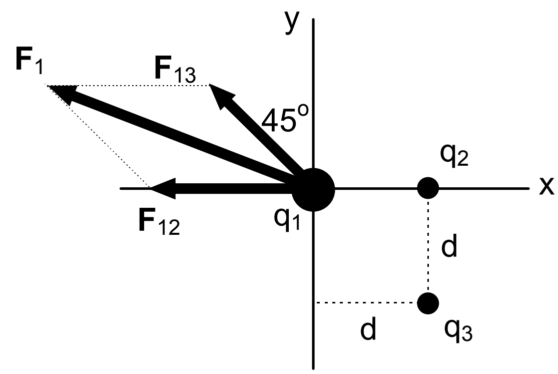
- Graphically. So, if you are given vectors **A** and **B**, and you want **C** = **A** + **B**, then put the *tail* of **B** onto the *head* of **A**, and **C** will be the vector going from the tail of **A** to the head of **B**.



- Break both vectors into components, and add. So, for **C** = **A** + **B**, we would have:

$$\mathbf{C} = C_x\mathbf{i} + C_y\mathbf{j} \quad \text{with} \quad C_x = A_x + B_x$$
$$C_y = A_y + B_y.$$

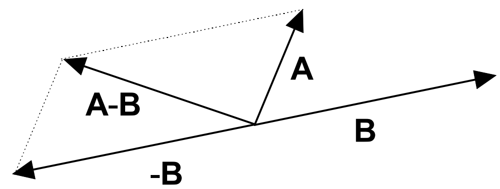
  Now all we need is A_x, A_y, etc. What do we choose for the x and y directions to take components along? *Whatever makes the problem easiest!* Let's say we have charges $+q_1$, $+q_2$, and $+q_3$ that are fixed in place as shown. The individual forces on q_1 are also shown. ($\mathbf{F}_{12}$ means "the force on charge 1 due to charge 2").
  What's the resultant force $\mathbf{F}_1$? Well, $\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13}$. Using Coulomb's law, we know that $\mathbf{F}_{12}$ points in the -x direction, and $\mathbf{F}_{13}$ points 45 degrees with respect to the y axis as shown in the figure.
  Therefore, we can write:



$$\mathbf{F}_{12} = -F_{12}\,\mathbf{i}$$
$$\mathbf{F}_{13} = -F_{13}\sin 45^\circ\,\mathbf{i} + F_{13}\cos 45^\circ\,\mathbf{j}$$
$$\mathbf{F}_1 = -(F_{13}\sin 45^\circ + F_{12})\,\mathbf{i} + F_{13}\cos 45^\circ\,\mathbf{j}$$

Subtracting vectors:

Take the same approach as when adding. It sometimes helps to think of **A** - **B** as **A** + (-**B**).



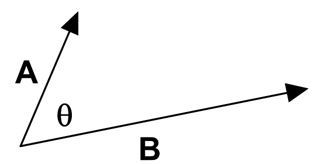
Dot products. Also called scalar products:

Note scalar product. The result of $\vec{A} \bullet \vec{B}$ is a number (scalar), not a vector. There are two ways to calculate the scalar product.

⇒ $\vec{A} \bullet \vec{B} = A_xB_x + A_yB_y$

⇒ $\vec{A} \bullet \vec{B} = AB\cos\theta$

In this last, θ is the angle between $\vec{A}$ and $\vec{B}$ when they are drawn *tail to tail*, as shown. Here are some important facts:

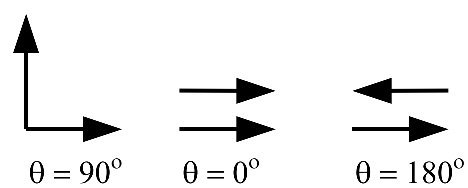


⇒ $\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$

⇒ $\vec{A} \bullet \vec{B} = 0$ when θ=90°.

⇒ $\vec{A} \bullet \vec{B} = AB$ when θ=0°. Dot product is a maximum.

⇒ $\vec{A} \bullet \vec{B} = -AB$ when θ=180°. Dot product is a minimum.



A little trigonometry:

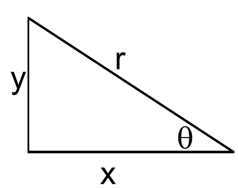
For the right triangle shown:

$\sin\theta = y/r$

$\cos\theta = x/r$

$\tan\theta = y/x$

$r = (x^2 + y^2)^{1/2}$



sin0° = 0	sin90° = 1	sin180° = 0	sin270° = -1
cos0° = 1	cos90° = 0	cos180° = -1	cos270° = 0

There is much more to the subject of vectors than what is covered on this handout, but the things given here are all material that should be *committed to memory* for this course, especially if you plan to pursue a career in a technical field such as engineering. The vector cross-product will be covered later when we get to magnetic field phenomena.