

## LAB 5

The goal of this lab is to become familiar with the concept of energy resolution and to measure the energy resolution of a NaI detector. In a previous lab we measured the response of the NaI detector to gamma rays and saw that a gamma ray shows up as a "peak" or "bump" in the MCA (Multi Channel Analyzer) spectrum. The shape of the peak is to a very good approximation well described by a Gaussian function. The mean of this Gaussian corresponds to the average energy of the gamma ray while the width of the Gaussian tells us something about how accurately our NaI detector measures the energy of the gamma ray.

The standard definition of energy resolution ( $R$ ) is:

$$R = \frac{\text{Full Width Half Maximum}}{\text{Position of Peak}}$$

For a Gaussian distribution the position of the peak = mean, and FWHM =  $2.36\sigma$ .

(I) To get a feeling for the spread in gamma ray energy from a relatively long lived isotope estimate the natural line width ( $\Delta E$ ) of Cs137 using the Heisenberg Uncertainty Principle:

$$\Delta E \Delta t \geq \hbar$$

with  $\hbar = 6.58 \times 10^{-16} \text{ eV} \cdot \text{sec}$ ,  $\Delta t$  taken to be the half-life of Cs137, 30 years.

Using your energy calibration curve from last week roughly how many voltage bins does this energy spread cover? Could you possibly ever see this energy spread using the equipment that we have in this lab?

(II) Measure the energy resolution of your NaI detector using Co60, Na22, and Cs137.

Take a spectrum of each sample and save the data run to disk. Note: Make sure that you accumulate enough data for each sample, i.e. wait (5 minutes) until the spectrum has "smoothed" itself out.

(a) *Crude calculation of energy resolution:*

Calculate the energy resolution using the voltages from your spectrum. For each line estimate the FWHM in terms of the voltage. Also estimate the peak voltage and calculate the energy resolution ( $R$ ) for the five gamma rays. Make a plot of your energy resolution (y-axis) vs. the true energy (x-axis).

(b) *A better estimate of the energy resolution:*

Calculate the energy resolution by first converting the voltage into energy using your calibration curve from last week. For each line estimate the Full Width at Half Maximum in terms of the energy spread and using the known energy of each decay calculate the energy resolution for the five gamma rays. Why is this a better way of estimating the energy resolution compared to (a)? Overlay the resolution on the plot from (a).

(c) *The ultimate calculation of the energy resolution:*

Fit your data to a function that represents the measured gamma ray spectrum. For the voltage bins near the gamma ray lines the spectrum can be thought to consist of two components, the gamma ray line itself (our signal), which we can model with a Gaussian, and background. If there was no background then the estimated number of counts in bin  $i$  would be  $P_i^S(n)$ :

$$P_i^S(n) = \frac{A}{\sigma} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}}$$

Here  $x_i$  is either the voltage or energy. We would like to determine the constants  $A$ ,  $\mu$ , and  $\sigma$  that best describe our data. Unfortunately, we also have background in our spectrum. Let's assume that the background (i.e. what we would have if we took the radioactive source away) in the area of interest can be modeled by a straight line (is this reasonable?):

$$P_i^B(n) = C + Dx_i$$

Again, the constants  $C$  and  $D$  are to be determined from our data. Now, since our data really consists of background and signal the best representation of our data is given by the sum of the signal and background functions:

$$P_i^{S+B}(n) = \frac{A}{\sigma} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}} + C + Dx_i$$

The problem now is to determine the 5 constants ( $A$ ,  $C$ ,  $D$ ,  $\mu$ ,  $\sigma$ ) from our data points. In principle this is simple, we use a technique such as Maximum Likelihood or Least Squares to find the 5 parameters. In practice this is difficult since the above function is non-linear in some of the variables, thus we would have to solve a set of 5 coupled non-linear equations! Rather than write a computer program to do this (you can if you want), we will use a feature of Kaleidagraph that fits complicated functions to data. Review the handout A Quick Guide to Kaleidagraph or read Chapter 9 of the Kaleidagraph Learning Guide to learn how to fit your data to a function and then fit your 5 gamma-ray lines (actually 4  $\gamma$ 's and a positron annihilation) to the above function. You should weight each data point using its statistical error (check the Weight Data option under Define when using Curve Fit). What are the  $\chi^2$  of the fits? Are they reasonable?

Note: Kaleidagraph will try to fit all of the data points in a file to the function of interest. In most cases however we only want to fit a small part of the spectrum (e.g. 50 voltage bins out of 250) to the function. Therefore it is best to copy the data in the region of interest to a new file and work with this file while performing the fits.

For each gamma ray line calculate the energy resolution using the mean ( $\mu$ ) and standard deviation ( $\sigma$ ) obtained from your fit. Don't forget to convert standard deviation into FWHM (you could fit for the FWHM instead of  $\sigma$ )! Overlay the resolution on the plot from (a) Is the energy resolution constant as a function of energy?

III) Simulate, using a computer, the performance of a NaI detector. In this part of the lab we would like to get a feeling for what a Co60 gamma ray spectrum would look like if the NaI detector had a percent energy resolution ( $R$ ) of 3%, 6%, or 25%. To do this you will write a program that simulates the response of a detector with a known energy resolution. In this exercise assume that the response of the detector is given by a Gaussian function with known mean ( $\mu$ ) and standard deviation ( $\sigma$ ). When Co60 decays it gives off two distinct gamma rays (not always, but 99% of the time), a lower energy gamma with  $E_l = 1.172$  MeV and a higher energy gamma ray with  $E_u = 1.333$  MeV. Let's represent the response of the NaI detector as a Gaussian with  $\mu = E_l$  or  $E_u$  depending on which gamma ray we are detecting. Using the

definition of energy resolution ( $R$ ), and defining  $E$  as the energy of the gamma ray the  $\sigma$  of the Gaussian is given by:

$$\sigma = \frac{E \times R}{2.36}$$

From a previous lab we know how to generate a Gaussian distribution using random numbers. Recall that the Gaussian you generated had  $\mu = 0$  and  $\sigma = 1$ . However for this lab we want a Gaussian with  $\sigma = E_1$  or  $E_2$  and  $\sigma$  given by the energy resolution. To transform (apart from a normalization constant) from a set of numbers  $\{x_1 \dots x_n\}$  that are distributed according to a Gaussian distribution with  $\mu = 0$  and  $\sigma = 1$  to a set of numbers  $\{y_1 \dots y_n\}$  also Gaussian distributed but with  $\mu = m$  and  $\sigma = s$  use:

$$y_i = s \cdot x_i + m$$

*Using the definition of a Gaussian distribution show that the above transformation really does what it claims to do!*

Use the following prescription to simulate on the computer the energy response of the detector to Co60. The goal here is to make a histogram similar to the one displayed by the LABVIEW program MCA.

- a) Choose an energy resolution (first do 3%, then 6%, then 25%).
- b) Use your Gaussian number generator to obtain a set of  $x_i$ 's.
- c) Transform each  $x_i$  to the new Gaussian variable  $y_i$ . Remember  $s$  is the chosen energy resolution and  $m$  is the true energy of the gamma ray. The  $y$ 's now represent the measured energy of the gamma rays. For each random number you generate you need to decide if represents the low energy or high energy gamma ray. For the low energy gamma  $m = 1.172$ , for the high energy gamma  $m = 1.333$  MeV.
- d) Keep track of the number of measured gamma rays with energies in the interval  $[E, E+\Delta E]$ . The LABVIEW program uses a bin size of  $\Delta E \sim 15$  KeV. I suggest using 25 bins with this bin size, the first bin starting at  $E = 1.1$  MeV.

For each energy resolution generate  $10^4$  low energy gammas and  $10^4$  high energy gammas (this represents  $10^4$  decays of Co60). Make a histogram of your energy spectrum for each of the three energy resolutions. Which of the three histograms looks most like your actual Co60 spectrum? Comment on how easy/difficult it is to tell that Co60 decays to two gamma rays if the energy resolution of your detector is  $\sim 25\%$ .