Durkin
Phys 416

**Problem Set 4**
**Due Nov 23**

1) The number of counts/min detected by a NaI counter in the vicinity of a radioactive source is given below for several trials. The NaI detector counts both signal (*S*) and background (*B*) when it is operating. By some other independent method the corresponding background count rate (*B*) is also measured. Use the information given below to calculate the average count rate (cts/min) due to just the radioactive source (*S*). What is the uncertainty ($\sigma_S$) in the count rate due to the source? Remember counting processes such as this exercise are Poisson in nature.

| Trial | Cnts/min (*S* + *B*) | Cnts/min (*B* only) |
|---|---|---|
| 1 | 170 | 15 |
| 2 | 175 | 18 |
| 3 | 200 | 25 |
| 4 | 160 | 17 |
| 5 | 150 | 21 |

2) (*Barlow problem 7.4*) Show that if eight trials produce four successes, the 90% confidence interval for the intrinsic probability *P* is from 19 to 81%. Compare these results with those obtained from a Gaussian approximation to the binomial. Repeat the problem assuming there were three successes.

3) The following is the numbers of neutrino events detected in 10 second intervals by the IMB experiment on 23 February 1987 -- around which time the supernova S 1987a was first seen by experimenters:

| # events | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| # intervals | 1042 | 860 | 307 | 78 | 15 | 3 | 0 | 0 | 0 | 1 |
| Prediction | | | | | | | | | | |

(a) Ignoring the interval with nine events, compute the average number of background events expected in an interval and fill in the predictions on the number of intervals to observe the number of events listed in the table.
(b) Justify that the interval with nine events is consistent with supernova explosion by computing the $\chi^2$ per degree of freedom with or without the interval with nine events.

4) (*Barlow problem 8.4*) Ten temperatures are measured, each with an error of 0.2 K:

| |
|---|
| 10.2 10.4 9.8 10.5 9.9 9.8 10.3 10.1 10.3 9.9 |

It is suggested that they are all the same true value, differences being due to the measurement errors. Find the number of degrees of freedom and $\chi^2$. Using 10% as the minimum acceptable confidence level, are they from the same true value? How would things be different if the original suggestion were that they are all the same true value of 10.1 K?

5) A star is giving off bursts of neutrinos. A sample of 130 such neutrino bursts yielded an average burst duration of 20 μseconds with a sample standard deviation of 4 μseconds.

Assuming Gaussian statistics are appropriate for this problem, compute a 99% confidence interval for the true average duration of a neutrino burst from this star.

6) A certain radioactive material gives off mono-energetic gamma rays of an unknown energy. A sample of 100 such gamma rays is measured and found to have an average energy of 1 MeV. The standard deviation of the measuring device is known to be 0.04 MeV. Assuming Gaussian statistics are appropriate for this problem, compute a 99% confidence interval for the true energy of the emitted gamma ray. First calculate the "error in the mean" for the gamma rays. Next find for a symmetric interval the number of standard deviations ($\sigma$) that correspond to 99% of the area (Taylor Appendix A) of a Gaussian. Finally calculate the confidence interval.

7) A certain theory states that the angular distribution of the decay of an unstable particle should have a probability distribution of the form:

$$p(\cos\theta) = N(1 + \alpha\cos^2\theta)$$

Here both $N$ and $\alpha$ are constants. An experiment measures ten examples of the decay of this unstable particle and finds the following values of $\cos\theta$: (-0.05, -0.15, -0.25, -0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95). We wish to determine the value of $\alpha$ using the Maximum Likelihood Method.

a) Use the normalization condition for a probability distribution function to show that:

$$N = \frac{1}{2(1+\alpha/3)}$$

For this problem the limits on $\cos\theta$ are [-1, 1].

b) Write down the Likelihood Function for this problem.

c) Make a plot of the Likelihood Function vs. $\alpha$ for $-1.5 < \alpha < 1.5$. Use this plot to find the value of $\alpha$ that maximizes the Likelihood Function. You may want to write (e.g.) a program to do this part!