

Problem Set 3
Due Nov 7

1) Maximum Likelihood Method:

Given the following probability distribution function for the variable x and constant λ ,

$$p(x, \lambda) = \lambda e^{-x\lambda}$$

and three measurements of x ($x_1 = 3$ sec, $x_2 = 5$ sec, $x_3 = 4$ sec):

- Write down the likelihood function for this problem.
- Use the Maximum Likelihood Method to calculate λ .

2) *Barlow 6.2* A trolley moves along a track with a constant speed, which you need to measure. It passes through the point $d = 0$ at exactly $t = 0$. At certain fixed distances, determined by sensing devices on the track, the time is measured with an error of 0.1 sec. The results are:

| | | | | | | |
|--------------------|-----|-----|-----|-----|-----|-----|
| Time t (seconds) | 1.1 | 2.2 | 2.9 | 4.1 | 5.0 | 5.8 |
| Distance d (mm) | 10 | 20 | 30 | 40 | 50 | 60 |

Find the velocity and the χ^2 .

3) *Barlow 4.5* If a current is determined by measuring a voltage of 45 ± 1 V through a resistance of $900 \pm 10 \Omega$, what is its value and error?

4) We wish to determine the index of refraction (n) of a medium by measuring how much light bends when it travels from one medium to another. The law relating angles and indexes of refraction is Snell's Law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

Given the following measurements find n_2 and using propagation of errors the uncertainty of n_2 .

$$n_1 = 1.0000 \quad \theta_1 = 22.03 \pm 0.2 \text{ degrees} \quad \theta_2 = 14.45 \pm 0.2 \text{ degrees}$$

5) We wish to determine the acceleration due to gravity (g).

a) Given the following data and the relationship:

$$x = 0.5gt^2$$

use the least squares technique to find the best value of g . Assume the error in each x measurement is 0.01 m and the time is measured exactly.

| x (m) | t (s) |
|---------|---------|
| 0.05 | 0.1 |
| 0.44 | 0.3 |
| 1.23 | 0.5 |
| 2.40 | 0.7 |

- What is the value of the chi-square for this problem?
- How many degrees of freedom are there in this problem?
- Estimate the probability to get a chi-square per degree of freedom larger than what you obtain using your results from parts b) and c).

6) Two different experiments have measured the mass of the X boson. Experiment #1 measured 1.00 ± 0.01 gm while experiment 2 measured 1.04 ± 0.02 gm. What is the best estimate of the mass of the X boson if we combine the two experiments?

Calculate the χ^2 for the two measurements in this problem using:

$$\chi^2 = \sum_{i=1}^2 \frac{(m_i - m)^2}{\sigma_i^2}$$

with m_i the measurement from experiment i and σ_i the standard deviation of the measurement, and m the best estimate of the mass obtained by combining the two experiments. How many degrees of freedom are there in this problem? What's the probability of getting a value of χ^2 greater than or equal to the one above?