

Hints for Burgess and Moore, Chapter 4

Problem 4.1 W width at finite fermion mass

40 points: 10 points for each part

4.1.1

Express the matrix element as the product of three matrix elements: $\langle 0|W_\mu^-(x=0)|W^-(\vec{k}, \lambda)\rangle$, $\langle e^-(\vec{p}, \sigma_1)|\bar{e}_i(x=0)|0\rangle$, and $\langle \bar{\nu}_e(\vec{q}, \sigma_2)|\nu_j(x=0)|0\rangle$, where i and j are Dirac indices.

4.1.2

Express the product of spinors as the trace of a matrix. Then use the identities in Eqs. (4.16) and (4.17) for sums over spins to express the matrix in terms of Dirac matrices. Evaluate the trace using the trace identities in Eqs. (4.23)-(4.28). The result given in the book is correct only if the polarization 4-vector ϵ^μ is real. If it is complex, the expression in curly brackets should be

$$m_e m_\nu (g_V^2 - g_A^2) + (g_V^2 + g_A^2)[-p \cdot q + \epsilon \cdot p \epsilon^* \cdot q + \epsilon^* \cdot q \epsilon \cdot p] + 2i g_V g_A \epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta \epsilon_\mu \epsilon_\nu^*.$$

4.1.3

“Linearly polarized in the \vec{e} direction” means $\epsilon^\mu = (0, \vec{e})$ in the rest frame of the W^- . This polarization vector is real, so the $g_V g_A$ term is 0. After setting $m_\nu = 0$, only the $g_V^2 + g_A^2$ term remains.

In the W^- rest frame, the momentum 4-vectors are $k^\mu = (M_W, \vec{0})$, $p^\mu = (E_p, \vec{p})$, and $q^\mu = (p, -\vec{p})$. Show that the differential phase space in the W^- rest frame reduces to

$$(2\pi)^4 \delta^4(k - p - q) \frac{d^3 p}{(2\pi)^3 2E_p} \frac{d^3 q}{(2\pi)^3 2E_q} = \frac{p}{8\pi(E_p + p)} d\cos\theta,$$

where θ is the angle between \vec{p} and \vec{e} . Solve the energy delta function constraint $p + E_p = M_W$ to obtain p as a function of M_W and m_e .

Problem 4.2 Decay of the top quark

40 points: 10 points for each part

Problem 4.3 Gamma-matrix identities

20 points: 10 points for each part