Hints for Burgess and Moore, Chapter 2

**Problem 2.1** Anomaly cancellation and charge assignments
30 points: 10 points for the hypercharges
10 points for each anomaly cancellation condition

Express each of the hypercharges as a function of \( \epsilon \) and \( \delta \). (Note that the hypercharge of the Higgs field is \( +\frac{1}{2} \), not \( -\frac{1}{2} \).) Express both of the anomaly cancellation conditions in terms of \( \epsilon \) and \( \delta \). Then solve for \( \epsilon \) and \( \delta \).

**Problem 2.2** Muon decay
10 points

**Problem 2.3** Right-handed neutrinos
40 points: 10 points for each part

2.3.1 First write down the new renormalizable terms for a single generation. Then generalize it to 3 generations. I have not yet figured out why the term \( LP_L N \phi \) is not allowed. If you figure it out, please explain it to me.

**Problem 2.4** Two Higgs doublet models
50 points: 10 points for each part

2.4.1 Use \( Q = T_3 + Y \).

2.4.3 The invariant \( c = \phi^T \epsilon \psi \) is complex. Its complex conjugate can be written \( c^* = \psi^\dagger \tilde{\phi} \).

2.4.4 The conditions for the electromagnetic \( U(1) \) symmetry to be unbroken are
(a) \( e^{i\theta Q} \phi_{\text{min}} = \phi_{\text{min}} \) (or \( Q \phi_{\text{min}} = 0 \)), where \( Q \) is the 2 x 2 charge matrix for \( \phi \),
(b) \( e^{i\theta Q} \psi_{\text{min}} = \psi_{\text{min}} \) (or \( Q \psi_{\text{min}} = 0 \)), where \( Q \) is the 2 x 2 charge matrix for \( \psi \).

The kinetic term for the scalar fields is

\[
\mathcal{L}_{\text{kinetic}} = -(D^\mu \phi)^* D_\mu \phi - (D^\mu \psi)^* D_\mu \psi
\]

The mass term \( \mathcal{L}_{\text{mass}} \) for the gauge bosons is obtained by setting \( \phi = \phi_{\text{min}} \) and \( \psi = \psi_{\text{min}} \). The masses of the \( W^\pm \) and \( Z^0 \) bosons are obtained by expressing it in the form

\[
\mathcal{L}_{\text{mass}} = -M_W^2 W^{-\mu} W^\mu + \frac{1}{2} M_Z^2 Z^\mu Z_\mu
\]
Problem 2.5  Adjoint Higgs field
30 points: 10 points for each part

2.5.3 Follow the same strategy as in Problem 2.4.

Problem 2.6  Gauged $B – L$ coupling
50 points: 10 points for each part

Problem 2.7  Colored scalar fields
Skip it!

Problem 2.8  Adjoint representation fermions
30 points: 10 points for each part