

Hints for Burgess and Moore, Chapter 2

Problem 2.1 Anomaly cancellation and charge assignments

30 points: 10 points for the hypercharges

10 points for each anomaly cancellation condition

Express each of the hypercharges as a function of ϵ and δ . (Note that the hypercharge of the Higgs field is $+\frac{1}{2}$, not $-\frac{1}{2}$.) Express both of the anomaly cancellation conditions in terms of ϵ and δ . Then solve for ϵ and δ .

Problem 2.2 Muon decay

10 points

Problem 2.3 Right-handed neutrinos

40 points: 10 points for each part

2.3.1 First write down the new renormalizable terms for a single generation. Then generalize it to 3 generations. I have not yet figured out why the term $\bar{L}P_L N \phi$ is not allowed. If you figure it out, please explain it to me.

Problem 2.4 Two Higgs doublet models

50 points: 10 points for each part

2.4.1 Use $Q = T_3 + Y$.

2.4.3 The invariant $c = \phi^T \epsilon \psi$ is complex. Its complex conjugate can be written $c^* = \psi^\dagger \tilde{\phi}$.

2.4.4 The conditions for the electromagnetic $U(1)$ symmetry to be unbroken are

(a) $e^{i\theta Q} \phi_{\min} = \phi_{\min}$ (or $Q \phi_{\min} = 0$), where Q is the 2 x 2 charge matrix for ϕ ,

(b) $e^{i\theta Q} \psi_{\min} = \psi_{\min}$ (or $Q \psi_{\min} = 0$), where Q is the 2 x 2 charge matrix for ψ .

The kinetic term for the scalar fields is

$$\mathcal{L}_{\text{kinetic}} = -(D^\mu \phi)^* D_\mu \phi - (D^\mu \psi)^* D_\mu \psi$$

The mass term $\mathcal{L}_{\text{mass}}$ for the gauge bosons is obtained by setting $\phi = \phi_{\min}$ and $\psi = \psi_{\min}$. The masses of the W^\pm and Z^0 bosons are obtained by expressing it in the form

$$\mathcal{L}_{\text{mass}} = -M_W^2 W^{-\mu} W_\mu^+ - \frac{1}{2} M_Z^2 Z^\mu Z_\mu$$

Problem 2.5 Adjoint Higgs field

30 points: 10 points for each part

2.5.3 Follow the same strategy as in Problem 2.4.

Problem 2.6 Gauged $B - L$ coupling

50 points: 10 points for each part

Problem 2.7 Colored scalar fields

Skip it!

Problem 2.8 Adjoint representation fermions

30 points: 10 points for each part