Collinear Photons in Z^0 Decay

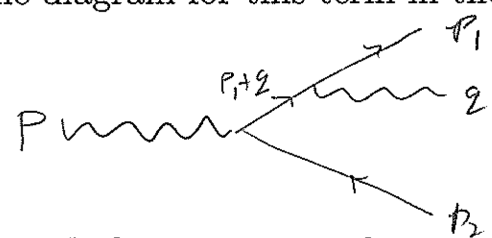
The matrix element $\mathcal{M}_1$ for the decay $Z^0(P) \to \mu^-(p_1)\mu^+(p_2)\gamma(q)$ is

$$\mathcal{M}_1 = e\,\varepsilon_\mu(P)\,\varepsilon^*_\alpha(q)\,\bar{u}(p_1)\left(\gamma^\alpha \frac{1}{\not{p}_1 + \not{q} - m_\mu}(g_V - g_A\gamma_5)\gamma^\mu \right.$$
$$\left. -(g_V - g_A\gamma_5)\gamma^\mu \frac{1}{\not{p}_1 + \not{q} - m_\mu}\gamma^\alpha\right) v(p_2).$$

In the region where μ^- and γ are nearly collinear, $\mathcal{M}_1$ in the limit $m_\mu \to 0$ can be approximated by

$$\mathcal{M}_1 \approx e\,\varepsilon_\mu(P)\,\varepsilon^*_\alpha(q)\,\bar{u}(p_1)\gamma^\alpha \frac{\not{p}_1 + \not{q}}{2p_1.q}(g_V - g_A\gamma_5)\gamma^\mu v(p_2).$$

A. Draw the diagram for this term in the matrix element, labelling the momenta.



The differential phase space can be expressed in an iterated form with additional integrals over the jet 3-momentum $\vec{P}_1 = \vec{p}_1 + \vec{q}$ and the jet invariant mass $P_1^2 = (p_1 + q)^2$:

$$d\Pi = \frac{d(P_1^2)}{2\pi}(2\pi)^4\delta^4(P - P_1 - p_2)\frac{d^3P_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2p_2}$$
$$\times (2\pi)^4\delta^4(P_1 - p_1 - q)\frac{d^3p_1}{(2\pi)^3 2p_1}\frac{d^3q}{(2\pi)^3 2q}$$

B. Verify this by using the identity

$$\int \frac{d^4P_1}{(2\pi)^4} 2\pi\delta(P_1^2 - M^2) = \int \frac{d^3P_1}{(2\pi)^3}\frac{1}{2\sqrt{\vec{P}_1^2 + M^2}}.$$

$$\int \frac{d^4P_1}{(2\pi)^4} \underbrace{\boxed{2\pi\,\delta(P_1^2 - M^2)\frac{dP_1^2}{2\pi}}}_{=1} (2\pi)^4\delta^4(P - P_1 - p_2)(2\pi)\delta^4(P_1 - p_1 - q) = (2\pi)^4\delta^4(P - p_1 - p_2 - q)$$

C. $\sum_{\text{spins}} |\mathcal{M}_1|^2$ has an explicit factor of $1/(2p_1.q)^2$. Show that if it is actually proportional to $1/(2p_1.q)$, the integral over P_1^2 diverges logarithmically as $m_\mu \to 0$.

$$\int_{m_\mu^2}^{Q^2} \frac{dP_1^2}{P_1^2} = \log\frac{Q^2}{m_\mu^2}$$

The sums over electron and photon spins are given by

$$\sum_{\text{spins}} u(p_1)\bar{u}(p_1) = \not{p}_1, \qquad \sum_{\text{spins}} \varepsilon^*_\alpha(q)\varepsilon_\beta(q) = -g_{\alpha\beta} + \frac{q_\alpha \bar{q}_\beta + \bar{q}_\alpha q_\beta}{q.\bar{q}},$$

where $\bar{q} = (|\vec{q}|, -\vec{q})$ if $q = (|\vec{q}|, \vec{q})$.

D. Use the collinear approximations $p_1 \approx z(p_1+q)$ and $q \approx (1-z)(p_1+q)$ to express the ratio $p_1.\bar{q}/q.\bar{q}$ as a function of z.

$$\frac{p_1\cdot\bar{q}}{q\cdot\bar{q}} \simeq \frac{z\,(p_1+q)\cdot\bar{q}}{(1-z)\,(p_1+q)\cdot\bar{q}} = \frac{z}{1-z}$$

In the square of the matrix element, the product of the Dirac factor associated with the μ^- line and the photon wavefunction factor is

$$\varepsilon^*_\alpha(q)\varepsilon_\beta(q)\,(\not{p}_1+\not{q})\gamma^\beta u(p_1)\bar{u}(p_1)\gamma^\alpha(\not{p}_1+\not{q}).$$

E. Use the sum over electron spins to simplify the Dirac factor.

$$(\not{p}_1+\not{q})\,\gamma^\beta \sum_{\text{spins}} u(p_1)\bar{u}(p_1)\gamma^\alpha(\not{p}_1+\not{q}) = (\not{p}_1+\not{q})\,\gamma^\beta\, \not{p}_1\, \gamma^\alpha\,(\not{p}_1+\not{q})$$

The contribution from the term $-g_{\alpha\beta}$ in the sum over photon spins is

$$(\not{p}_1+\not{q})\big(-\gamma_\alpha\not{p}_1\gamma^\alpha\big)(\not{p}_1+\not{q}).$$

F. Use the Dirac identities $\gamma_\alpha \not{a}\gamma^\alpha = -2\not{a}$ and $\not{a}\not{b}\not{a} = 2a.b\,\not{a} - a^2\not{b}$ to reduce this in the collinear limit to $2p_1.q\,(\not{p}_1+\not{q})$ multiplied by a function of z.

$$= (\not{p}_1+\not{q})\,2\not{p}_1\,(\not{p}_1+\not{q}_1) = 2\left[2\,p_1\cdot(p_1+q)\,(\not{p}_1+\not{q}) - (p_1+q)^2\not{p}_1\right]$$

$$= 2\left[2p_1\cdot q\,(\not{p}_1+\not{q}) - 2p_1\cdot q\,\not{p}_1\right] = 4p_1\cdot q\,\not{q} \approx 4p_1\cdot q\,(1-z)(\not{p}_1+\not{q})$$

The contribution from the second term in the sum over photon spins is

$$\frac{1}{q.\bar{q}}(\not{p}_1+\not{q})\big(\not{q}\not{p}_1\not{\bar{q}} + \not{\bar{q}}\not{p}_1\not{q}\big)(\not{p}_1+\not{q}).$$

G. Use the Dirac identities $\not{a}\not{b}\not{c} + \not{c}\not{b}\not{a} = 2(a.b\,\not{c} - a.c\,\not{b} + b.c\,\not{a})$ and $\not{a}\not{b}\not{a} = 2a.b\,\not{a} - a^2\not{b}$ to reduce this in the collinear limit to $2p_1.q\,(\not{p}_1+\not{q})$ multiplied by a function of z.

$$\frac{1}{q\cdot\bar{q}}(\not{p}_1+\not{q})\,2\left[q\cdot p_1\,\not{\bar{q}} - q\cdot\bar{q}\,\not{p}_1 + p_1\cdot\bar{q}\,\not{q}\right](\not{p}_1+\not{q}_1)$$

$$= \frac{2}{q\cdot\bar{q}}\Big(2\left[q\cdot p_1\,\bar{q}\cdot(p_1+q) - q\cdot\bar{q}\,p_1\cdot(p_1+q) + p_1\cdot\bar{q}\,q\cdot(p_1+q)\right](\not{p}_1+\not{q}) - (p_1+q)^2\left[q\cdot p_1\not{\bar{q}} - q\cdot\bar{q}\,\not{p}_1 + p_1\cdot\bar{q}\,\not{q}\right]\Big)$$

$$= \frac{2}{q\cdot\bar{q}}\Big(2p_1\cdot q\left[\bar{q}\cdot p_1 + \cancel{\bar{q}\cdot q} - \cancel{q\cdot\bar{q}} + p_1\cdot\bar{q}\right]\cdot(\not{p}_1+\not{q}_1) - 2p_1\cdot q\left[\cancelto{0}{q\cdot p_1}\,\not{\bar{q}} - q\cdot\bar{q}\,\not{p}_1 + p_1\cdot\bar{q}\,\not{q}\right]\Big)$$

$$= 4p_1\cdot q\left(2\frac{p_1\cdot\bar{q}}{q\cdot\bar{q}}(\not{p}_1+\not{q}_1) + \not{p}_1 - \frac{p_1\cdot\bar{q}}{q\cdot\bar{q}}\,\not{q}\right) = 4p_1\cdot q\,(\not{p}_1+\not{q})\left[2\frac{z}{1-z} + z - \frac{z}{1-z}(1-z)\right]$$