

Thornton and Marion

5-3.

The total energy of a particle with velocity v at a distance r from the center of the earth is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

A particle with the minimum energy to escape to $r = \infty$ will have $E = 0$. Thus the escape velocity must satisfy

$$0 = \frac{1}{2}mv_{esc}^2 - \frac{GMm}{R_e}$$

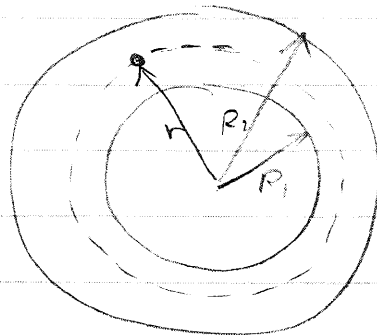
where R_e is the radius of the earth. The solution is

$$\begin{aligned}v_{esc} &= \sqrt{\frac{2GM}{R_e}} = \sqrt{2 \frac{GM}{R_e^2} R_e} \\&= \sqrt{2gR_e} \\&= \sqrt{2(9.8 \frac{m}{s^2})(6.38 \times 10^6 m)} \\&= 1.12 \times 10^4 \frac{m}{s} \\&\approx 25000 \frac{mile}{hour}\end{aligned}$$

Thornton and Marion

5-13.

Suppose the particle is at a radius r from the center of the planet, where $R_1 < r < R_2$. There is no net



force from the spherical shell of dust in the region beyond the radius of the particle. The net force from the dust inside the radius r and the planet is proportional to the total mass inside the radius r :

$$M(r) = \rho_1 \left(\frac{4}{3} \pi R_1^3 \right) + \rho_2 \left(\frac{4}{3} \pi (r^3 - R_1^3) \right)$$

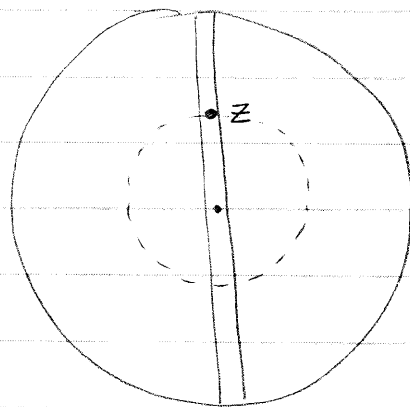
The total force on the planet is towards the center of the planet with magnitude

$$\begin{aligned} F(r) &= \frac{GM(r)m}{r^2} \\ &= \frac{Gm}{r^2} \left[\frac{4\pi}{3} (\rho_1 - \rho_2) R_1^3 + \frac{4\pi}{3} \rho_2 r^3 \right] \\ &= \frac{4\pi}{3} Gm \left[\frac{(\rho_1 - \rho_2) R_1^3}{r^2} + \rho_2 r \right] \end{aligned}$$

Thornton and Marion

5-15.

We choose the z -axis to be along the direction of the hole. When the particle has coordinate, the net gravitational force from particles in the earth with radius $r > |z|$ is zero. The net gravitational force from particles in the earth with radius $r < |z|$ acts as if all the mass inside that radius is at the center of the earth. The total mass inside the radius $r = |z|$ is



$$M(z) = \rho \cdot \frac{4}{3} \pi |z|^3$$

Newton's equation for the z -coordinate is

$$\begin{aligned} m \ddot{z} &= - \operatorname{sign}(z) \frac{GM(z)m}{z^2} \\ &= - \operatorname{sign}(z) \frac{Gm\rho \frac{4}{3} \pi |z|^3}{z^2} \\ &= - \frac{4\pi}{3} Gm\rho z \end{aligned}$$

This is the equation for a simple harmonic oscillator with angular frequency

$$\omega_0 = \sqrt{\frac{4\pi}{3} G\rho}$$

The period is

$$T = \frac{2\pi}{\omega_0} = \sqrt{\frac{3\pi}{G\rho}}$$

The density of the earth is

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

where M and R are the mass and radius of the earth.
Then the period is

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}} = \sqrt{\frac{4\pi^2 R}{g}}$$

$$= \sqrt{\frac{4\pi^2 (6.38 \cdot 10^6 \text{ m})}{9.8 \text{ m/s}^2}}$$

$$= 5.07 \times 10^3 \text{ s}$$

$$\approx 84.5 \text{ minutes}$$

Thornton and Marion

5-19

Let t be the time after a specific high tide.

In that time, the moon will revolve by an angle θ given by

$$\theta = 2\pi \frac{t}{T_{\text{moon}}}$$

where $T_{\text{moon}} = 27.3$ days. For the next high tide to occur at time t , the earth must revolve by the angle $\pi + \theta$;

$$\pi + \theta = 2\pi \frac{t}{T_{\text{earth}}}$$

where $T_{\text{earth}} = 24$ hr. Eliminating θ , we get

$$\pi = 2\pi \left(\frac{t}{T_{\text{earth}}} - \frac{t}{T_{\text{moon}}} \right)$$

$$t = \frac{\frac{1}{2}}{\frac{1}{T_{\text{earth}}} - \frac{1}{T_{\text{moon}}}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{(24 \text{ hr})} - \frac{1}{(27.3 \times 24 \text{ hr})}}$$

$$= 12.46 \text{ hours} \approx 12 \text{ hrs} + 27 \text{ min.}$$

