Thornton and Marion

2.17

The equations for the coordinate are

$$x(t) = v_0 \cos\theta \, t$$

$$y(t) = y_0 + v_0 \sin\theta \, t - \frac{1}{2} g t^2$$

where $y_0 = 0.7m$ and $\theta = 35°$.

We need $y(t) > y_1 = 2m$ when $x(t) = x_1 = 60m$

This happens at time $t = \frac{x_1}{v_0 \cos\theta}$

$$y_1 = y_0 + v_0 \sin\theta \frac{x_1}{v_0 \cos\theta} - \frac{1}{2} g \left(\frac{x_1}{v_0 \cos\theta}\right)^2$$

$$= y_0 + x_1 \tan\theta - \frac{g x_1^2}{2 v_0^2 \cos^2\theta}$$

Solve for v_0:

$$v_0 = \sqrt{\frac{g x_1^2}{2\cos^2\theta \left[y_0 - y_1 + x_1 \tan\theta\right]}}$$

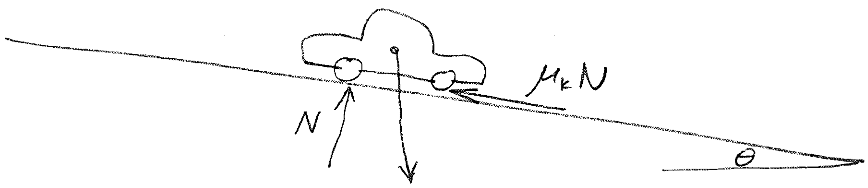
$$= \sqrt{\frac{(9.8 m/s^2)(60m)^2}{2\cos^2 35° \left[(60m)\tan 35° - 1.3m\right]}}$$

$$= 25.4 \frac{m}{s}$$

Thornton and Marion

2.29

The forces on the car after the driver hits the breaks are gravity (mg downward) and friction ($\mu_K N$ along the incline where N is the normal force).



The normal force balances the component of gravity perpendicular to the surface

$$N = mg\cos\theta$$

The component of the gravity force along the incline is $mg\sin\theta$. The equation for motion along the incline is therefore

$$m\ddot{x} = mg\sin\theta - \mu_K mg\cos\theta$$

$$\ddot{x} = -(\mu_K\cos\theta - \sin\theta)g$$

If the initial velocity is v_0, the velocity at time t is

$$\dot{x}(t) = v_0 - (\mu_K\cos\theta - \sin\theta)gt$$

The car comes to a stop after time

$$T = \frac{v_0}{(\mu_k \cos\theta - \sin\theta) g}$$

The distance travelled by the car in time t is

$$x(t) = v_0 t - \frac{1}{2}(\mu_k \cos\theta - \sin\theta) g t^2$$

After time T, the car has moved a distance

$$x(T) = v_0 T - \frac{1}{2}(\mu_k \cos\theta - \sin\theta) g T^2$$

$$= \frac{v_0^2}{2(\mu_k \cos\theta - \sin\theta) g}$$

The speed limit is $25 \frac{\text{miles}}{\text{hr}} \times \frac{1609\,\text{m}}{\text{mi}} \times \frac{\text{hr}}{3600\,\text{s}} = 11.2\,\text{m/s}$.
The incline has an 8% grade, so $\tan\theta = 0.08$, $\theta = 0.0798$.
The kinetic coefficient of friction is measured to be $\mu_k = 0.45$
If the car was going the speed limit, the distance it would travel before stopping is

$$\frac{v_0^2}{2(\mu_k \cos\theta - \sin\theta) g} = \frac{(11.2\,\text{m/s})^2}{2(0.45 \cos\theta - \sin\theta)\, 9.8\,\text{m/s}^2}$$

$$= 17.3\,\text{m}$$

If the car skidded 30 m, it must have been going faster than 25 mi/hr.

Thornton and Marion

2.41

(a) As measured by a person on the train, the velocity of the ball increase from 0 to v, so the gain in kinetic energy is

$$\Delta T = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$$

(b) As measured by a person on the railroad track, the velocity of the ball increases from u to $u+v$, so the increase in kinetic energy is

$$\Delta T = \frac{1}{2}m(u+v)^2 - \frac{1}{2}mu^2$$

$$= \frac{1}{2}mv^2 + muv$$

(c) The work done by the woman throwing the ball is ΔT

(d) The work done by the train is 0.

For the work done by the woman on the ball to give $\Delta T = \frac{1}{2}mv^2$ in the frame of the woman, the force applied to the ball as a function of time must satisfy

$$\int F(t)\,\dot{x}(t)\,dt = \tfrac{1}{2}mv^2$$

The same integral without the factor of $\dot{x}$ must give the change in momentum

$$\int F(t)\,dt = \int \frac{dp}{dt}\,dt = mv - 0 = mv$$

In the frame of the railroad tracks, the work done by the woman on the ball is

$$\int F\,dx = \int F(t)\left[\dot{x}(t) + u\right] dt$$

$$= \int F(t)\,\dot{x}(t)\,dt + u\int F(t)\,dt$$

$$= \tfrac{1}{2}mv^2 + u\cdot mv$$

$$= \Delta T$$

Thornton and Marion

2-55

The equations of motion are

$$m\ddot{\vec{r}} = -mg\hat{z} - kmv\hat{v}$$

$$= -mg\hat{z} - km\dot{\vec{r}}$$

The x and z components are

$$\ddot{x} = -k\dot{x}$$

$$\ddot{z} = -g - k\dot{z}$$

The initial conditions were

$x(0) = 0$ $\qquad\qquad$ $\dot{x}(0) = v_0 \cos\theta = \frac{1}{\sqrt{2}} v_0$

$z(0) = 0$ $\qquad\qquad$ $\dot{z}(0) = v_0 \sin\theta = \frac{1}{\sqrt{2}} v_0$

We first solve the x equation

$$\frac{\ddot{x}}{\dot{x}} = -k$$

$$(\ln \dot{x})^{\cdot} = -k$$

$$\ln \dot{x} - \ln \frac{v_0}{\sqrt{2}} = -kt$$

$$\dot{x}(t) = \frac{v_0}{\sqrt{2}} e^{-kt}$$

$$x(t) = \frac{v_0}{\sqrt{2}k}(1 - e^{-kt})$$

We next solve the z equation

$$\ddot{z} + k\dot{z} = -g$$

$$\left(\dot{z}e^{+kt}\right)^{\cdot} = -ge^{+kt}$$

$$\dot{z}e^{+kt} - \frac{v_0}{\sqrt{2}} = -\frac{g}{k}(e^{+kt} - 1)$$

$$\dot{z}(t) = \left(\frac{v_0}{\sqrt{2}} + \frac{g}{k}\right)e^{-kt} - \frac{g}{k}$$

$$z(t) = \left(\frac{v_0}{\sqrt{2}} + \frac{g}{k}\right)\frac{1}{k}(1 - e^{-kt}) - \frac{g}{k}t$$

The range R and the time T when the pumkin lands are related by

$$R = \frac{v_0}{\sqrt{2}k}(1 - e^{-kT})$$

$$0 = \left(\frac{v_0}{\sqrt{2}} + \frac{g}{k}\right)\frac{1}{k}(1 - e^{-kT}) - \frac{g}{k}T$$

We can solve the first equation for T:

$$T = -\frac{1}{k}\ln\left(1 - \frac{\sqrt{2}kR}{v_0}\right)$$

Eliminating T from the 2^{nd} equation, we get

$$0 = \left(\frac{v_0}{\sqrt{2}} + \frac{g}{k}\right)\frac{\sqrt{2}R}{v_0} - \frac{g}{k}\left[-\frac{1}{k}\ln\left(1 - \frac{\sqrt{2}kR}{v_0}\right)\right]$$

$$\ln\left(1 - \frac{\sqrt{2}kR}{v_0}\right) = -\frac{k^2}{g}R\left(1 + \frac{\sqrt{2}g}{v_0 k}\right)$$

Setting $R = 142\ m$

$v_0 = 54\ m/s$

$g = 9.8\ m/s^2$

and solving numerically for k, we get

$k = 5.7 \times 10^{-9}\ s^{-1}$