

The $c \rightarrow u\gamma$ Contribution to Weak Radiative Charm Decay

Christoph Greub^{1,2}

*Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309, USA*

Tobias Hurth^{1,3}, Mikołaj Misiak^{1,4} and Daniel Wyler¹

*Institut für Theoretische Physik der Universität Zürich,
Winterthurerstrasse 190, 8057 Zürich, Switzerland.*

Abstract

The $c \rightarrow u\gamma$ transition does not occur at the tree level in the Standard Model and is strongly GIM-suppressed at one loop. The leading QCD logarithms are known to enhance the amplitude by more than an order of magnitude. We point out that the amplitude increases further by two orders of magnitude after including the complete 2-loop QCD contributions. Nevertheless, $\Delta S = 0$ radiative decays of charmed hadrons remain dominated by the $c \rightarrow d\bar{d}u\gamma$ and $c \rightarrow s\bar{s}u\gamma$ quark subprocesses.

¹ Work supported in part by Schweizerischer Nationalfonds.

² Work supported in part by the Department of Energy, contract DE-AC03-76SF00515.

³ Address after March 1996: SUNY at Stony Brook, Stony Brook NY 11794-3840, USA.

⁴ Partially supported by the Committee for Scientific Research, Poland.

1. Inclusive heavy flavor decay can be systematically analyzed with help of an expansion in inverse powers of the heavy quark mass [1]. At the leading order in such an expansion, the inclusive hadronic decay rate is given by the perturbatively calculable free quark decay rate. For charmed hadron decays, this procedure can certainly be questioned since the charm quark mass is not really much larger than the QCD scale Λ . Nevertheless, some properties of charmed hadrons have been analyzed successfully with help of HQET [2]. Therefore, it is also of interest to calculate inclusive decay rates of these particles at the leading order in the heavy mass expansion.

In the present letter, we consider weak radiative charm decay [3, 4]. For definiteness, we restrict ourselves to $\Delta S = 0$ processes. At the leading order in electroweak and strong interactions, there are three contributions to the $\Delta S = 0$ charm quark radiative decay: $c \rightarrow u\gamma$, $c \rightarrow u d \bar{d} \gamma$ and $c \rightarrow u s \bar{s} \gamma$. At higher orders in the strong interactions, there are virtual corrections to these decays as well as new decay modes with more gluons and quark-antiquark pairs in the final state.

In the absence of QCD, the $c \rightarrow u\gamma$ amplitude is enormously suppressed by GIM cancellations and by small Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The leading logarithmic contribution turns out to be considerably larger, but is still afflicted with a small CKM coefficient. This raises a hope that higher order terms can be the most important ones, if they are not suppressed by small CKM factors.

In this paper, we show that the $c \rightarrow u\gamma$ process is completely dominated by a two-loop term which has not been considered so far in perturbative analyses of charm decays. Furthermore, we point out that decays with more gluons which are also of higher order in the strong interactions have comparable amplitudes.

The $c \rightarrow u\gamma$ channel cannot be separated kinematically from the accompanying modes by making a cut on the photon energy spectrum, because all the decay products are light. This is contrary to the $b \rightarrow s\gamma$ decay which can be separated from the $b \rightarrow s c \bar{u} \gamma$ and other similar channels by selecting photon energies above the charm production threshold. In the $c \rightarrow u\gamma$ case, we might still hope to be able to separate this channel by looking at the shape of photon energy spectrum. However, the possibility of doing so depends crucially on how large the $c \rightarrow u\gamma$ amplitude is.

2. We begin with the $c \rightarrow u\gamma$ process and briefly review the known contributions which come at the (formally) lower orders of perturbation theory. All momentum invariants in a two-body decay are expressible in terms of masses. Therefore, we can represent the amplitude by a tree diagram with a single local vertex. For the $c \rightarrow u\gamma$ decay in the Standard Model, the relevant local interaction reads

$$L_{int} = -\frac{4G_F}{\sqrt{2}} A \frac{e}{16\pi^2} m_c (\bar{u}\sigma_{\mu\nu}P_R c) F^{\mu\nu}, \quad (1)$$

where $P_R = (1+\gamma_5)/2$ and we have neglected the u -quark mass. The coefficient A nontrivially depends on light quark masses (see below). Thus, it should not be misinterpreted as a Wilson coefficient.

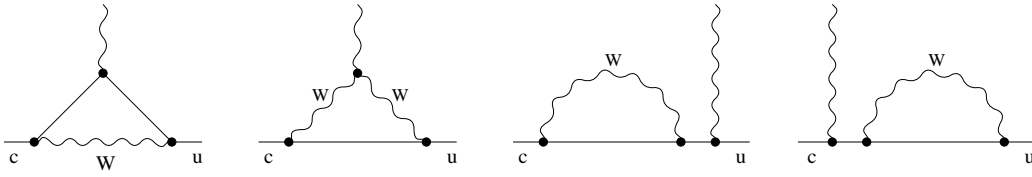


Figure 1: One-loop diagrams generating the $c \rightarrow u\gamma$ transition.

As any flavor changing neutral current process, the $c \rightarrow u\gamma$ amplitude arises in the Standard Model only at the one-loop level. The relevant diagrams are shown in fig. 1 and give the following contribution to the coefficient A :

$$\Delta A_{1 \text{ loop}} \simeq -\frac{5}{24} \sum_{q=d,s,b} V_{cq}^* V_{uq} \left(\frac{m_q}{M_W} \right)^2. \quad (2)$$

The CKM factors in the above equation have very different orders of magnitude

$$|V_{cd}^* V_{ud}| \simeq |V_{cs}^* V_{us}| \simeq 0.22 \quad \text{and} \quad |V_{cb}^* V_{ub}| \simeq (1.3 \pm 0.4) \times 10^{-4}. \quad (3)$$

Consequently, $|\Delta A_{1 \text{ loop}}| \sim 2 \times 10^{-7}$. The extraordinary smallness of this number is due to the tiny factors $(m_q/M_W)^2$ for the light quarks and to the small CKM angles in the b -quark contribution.

Since the important suppression factors are independent of gauge couplings, it is possible that higher orders in perturbation theory give dominant contributions to the radiative amplitude considered because they may not suffer the same dramatic suppressions and are reduced only by powers of the gauge couplings.

A natural first attempt to include higher orders is to resum short distance QCD corrections in the leading-logarithmic approximation, by analogy to the $b \rightarrow s\gamma$ decay where they

bring sizeable enhancements [5]. In order to systematically include these contributions, one introduces two effective hamiltonians

$$H_{eff}(M_W > \mu > m_b) = \frac{4G_F}{\sqrt{2}} \sum_{q=d,s,b} V_{cq}^* V_{uq} [C_1(\mu)O_1^q + C_2(\mu)O_2^q] \quad (4)$$

$$H_{eff}(m_b > \mu > m_c) = \frac{4G_F}{\sqrt{2}} \sum_{q=d,s} V_{cq}^* V_{uq} [C_1(\mu)O_1^q + C_2(\mu)O_2^q + \sum_{i=3}^8 C_i(\mu)O_i], \quad (5)$$

where

$$O_1^q = (\bar{u}_\alpha \gamma_\mu P_L q_\beta)(\bar{q}_\alpha \gamma^\mu P_L c_\beta), \quad q = d, s, b \quad (6)$$

$$O_2^q = (\bar{u}_\alpha \gamma_\mu P_L q_\alpha)(\bar{q}_\beta \gamma^\mu P_L c_\beta), \quad q = d, s, b \quad (7)$$

$$O_7 = \frac{e}{16\pi^2} m_c (\bar{u}_\alpha \sigma_{\mu\nu} P_R c_\alpha) F^{\mu\nu}. \quad (8)$$

The remaining operators O_i are given explicitly in eqn. (18) of ref. [3]. We do not present them here because they are inessential for our discussion.

As indicated in eqns. (4) and (5), each of the hamiltonians is valid in a different range of the renormalization scale μ . Now, we neglect all terms which are suppressed by additional powers of $1/M_W^2$ such as the ones in eqn. (2). Due to CKM unitarity, the operators O_3, \dots, O_8 are not generated by QCD renormalization at scales $\mu > m_b$.

Finding the coefficients $C_i(\mu = M_W)$ and performing the renormalization group evolution from $\mu = M_W$ to $\mu = m_c$ is by now standard [6]. Analogously to the $b \rightarrow s\gamma$ analysis of ref. [5], we introduce an ‘‘effective’’ anomalous dimension matrix. It reads

$$\hat{\gamma}^{(0)eff} = \begin{bmatrix} -2 & 6 & 0 & 0 & 0 & 0 & 0 & 3 \\ 6 & -2 & -\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} & 8Q_1 + \frac{16}{27}Q_2 & \frac{70}{27} \\ 0 & 0 & -\frac{22}{9} & \frac{22}{3} & -\frac{4}{9} & \frac{4}{3} & \frac{464}{27}Q_2 & \frac{140}{27} + 3f \\ 0 & 0 & 6 - \frac{2}{9}f & -2 + \frac{2}{3}f & -\frac{2}{9}f & \frac{2}{3}f & 8\bar{Q} + \frac{16}{27}fQ_2 & 6 + \frac{70}{27}f \\ 0 & 0 & 0 & 0 & 2 & -6 & -\frac{32}{3}Q_2 & -\frac{14}{3} - 3f \\ 0 & 0 & -\frac{2}{9}f & \frac{2}{3}f & -\frac{2}{9}f & -16 + \frac{2}{3}f & -8\bar{Q} + \frac{16}{27}fQ_2 & -4 - \frac{119}{27}f \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{32}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{32}{3}Q_2 & \frac{28}{3} \end{bmatrix}$$

For the renormalization group evolution from $\mu = m_b$ to $\mu = m_c$ in the $c \rightarrow u\gamma$ case, one needs to substitute $f = 4$, $Q_1 = Q_d = -\frac{1}{3}$, $Q_2 = Q_u = \frac{2}{3}$ and $\bar{Q} = 2Q_u + 2Q_d = \frac{2}{3}$.

For the evolution from $\mu = M_W$ to $\mu = m_b$ in the $b \rightarrow s\gamma$ case, one would need to substitute $f = 5$, $Q_1 = Q_u$, $Q_2 = Q_d$ and $\bar{Q} = 2Q_u + 3Q_d = \frac{1}{3}$. In this case, the matrix $\hat{\gamma}^{(0)eff}$ given in Appendix A of ref. [5] would be reproduced.

The resulting leading logarithmic contribution to A in eqn. (1) reads

$$\Delta A_{LLA} = -V_{cb}^* V_{ub} C_7^{eff}(m_c), \quad (9)$$

where

$$C_7^{eff}(m_c) = \sum_{i=1}^8 \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{z_i} [x_i C_1(m_b) + y_i C_2(m_b)]. \quad (10)$$

The coefficients $C_{1,2}(m_b)$ are given by

$$C_{1,2}(m_b) = \frac{1}{2} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{\frac{6}{23}} \mp \frac{1}{2} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{-\frac{12}{23}}, \quad (11)$$

and the numbers x_i , y_i and z_i read

$$\begin{aligned} x_i &= \left(-\frac{65710}{18413}, \frac{22173}{8590}, \frac{2}{5}, 0, 0.6524, -0.0532, -0.0034, -0.0084 \right), \\ y_i &= \left(-\frac{675158}{165717}, \frac{23903}{8590}, \frac{2}{5}, 0, 0.8461, 0.0444, 0.0068, -0.0059 \right), \\ z_i &= \left(\frac{14}{25}, \frac{16}{25}, \frac{6}{25}, -\frac{12}{25}, 0.3469, -0.4201, -0.8451, 0.1317 \right). \end{aligned} \quad (12)$$

Note that the r.h.s. of eqn. (10) vanishes in the formal limit $\alpha_s(m_c) \rightarrow \alpha_s(m_b)$, as it should.

Taking $m_b = 5$ GeV, $m_c = 1.5$ GeV and $\alpha_s(M_Z) = 0.12$ (which implies $\alpha_s(m_b)/\alpha_s(m_c) \simeq 0.67$), we obtain

$$|\Delta A_{LLA}| = [0.001C_1(m_b) + 0.055C_2(m_b)] |V_{cb}^* V_{ub}| = 0.060 |V_{cb}^* V_{ub}| \simeq (8 \pm 3) \times 10^{-6}. \quad (13)$$

This result is more than an order of magnitude larger than the (formally) leading order one in eqn. (2): Including logarithmic QCD contributions replaces the powerlike GIM suppression factors $(m_q/M_W)^2$ by a logarithmic function of m_b/m_c .

This surprising enhancement was noted some time ago and discussed in detail in ref. [3]. However, our coefficient $C_7^{eff}(m_c)$ in eqn. (10) is 5 times smaller than the one obtained there. This difference arises mainly because the operators O_1^b and O_2^b have not been taken into account in ref. [3]. Their presence above $\mu = m_b$ is essential for cancellation of additive logarithmic QCD contributions at these renormalization scales.

3. Although the leading logarithmic result (13) is much larger than the purely electroweak contribution (2), there remains a strong cancellation between s and d loops whose CKM factors are very similar in magnitude but have opposite signs. Consequently, a suppression by $V_{cb}^* V_{ub}$ is still present in eqn. (13).

This cancellation is circumvented when the functional dependence of the amplitude on the s - and d -quark masses becomes substantial. It turns out that this happens in the two-loop

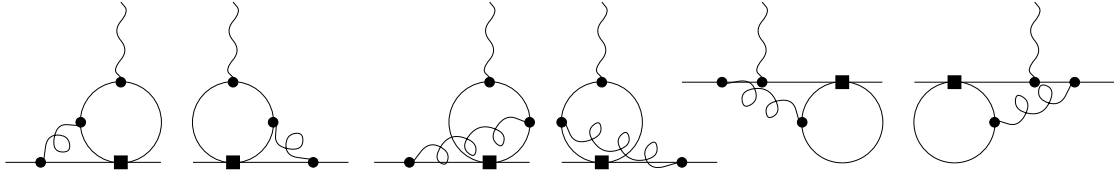


Figure 2: Diagrams contributing to the two-loop $c \rightarrow u\gamma$ matrix element of O_2^q .

matrix element of the effective hamiltonian in eqn. (5). It is given by the diagrams in fig. 2 with O_2^s and O_2^d operator insertions.²

After using unitarity of the CKM matrix, we obtain the following contribution to the coefficient A:

$$A = V_{cs}^* V_{us} \frac{\alpha_s(m_c)}{4\pi} C_2(m_c) \{f[(m_s/m_c)^2] - f[(m_d/m_c)^2]\} + \mathcal{O}(V_{cb}^* V_{ub}), \quad (14)$$

where

$$C_2(m_c) = \frac{1}{2} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{\frac{6}{23}} \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{\frac{6}{25}} + \frac{1}{2} \left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{-\frac{12}{23}} \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{-\frac{12}{25}}. \quad (15)$$

The function f can be extracted from an analogous computation performed recently for the b -quark decay [7]. It reads

$$\begin{aligned} f(z) = & -\frac{1}{243} \left\{ 576\pi^2 z^{3/2} \right. \\ & + \left[3672 - 288\pi^2 - 1296\zeta(3) + (1944 - 324\pi^2)L + 108L^2 + 36L^3 \right] z \\ & + \left[324 - 576\pi^2 + (1728 - 216\pi^2)L + 324L^2 + 36L^3 \right] z^2 \\ & + \left. \left[1296 - 12\pi^2 + 1776L - 2052L^2 \right] z^3 \right\} \\ & - \frac{4\pi i}{81} \left\{ \left[144 - 6\pi^2 + 18L + 18L^2 \right] z \right. \\ & + \left[-54 - 6\pi^2 + 108L + 18L^2 \right] z^2 \\ & + \left. \left[116 - 96L \right] z^3 \right\} + \mathcal{O}(z^4 L^4), \end{aligned} \quad (16)$$

where $L = \log z$. This function is renormalization scheme independent. All scheme dependent terms in the two-loop matrix elements of O_2^s and O_2^d undergo GIM cancellation and only affect the $\mathcal{O}(V_{cb}^* V_{ub})$ part of eqn. (14).

In the interesting range $0.005 < z < 0.1$, the function $f(z)$ is approximated (to 15% accuracy) by

$$f(z) \simeq -19z(1 - 2z) - 3iz \log^2 z. \quad (17)$$

² Two-loop diagrams with O_1^q insertions vanish due to their color structure. One-loop $c \rightarrow u\gamma$ diagrams with O_1^q or O_2^q insertions vanish for on-shell photons.

Using the s-quark mass $m_s(\mu = m_c) = (0.17 \pm 0.03)$ GeV [8] (but keeping $m_c = 1.5$ GeV fixed), we find

$$|A| = |V_{cs}^* V_{us}| \frac{\alpha_s(m_c)}{4\pi} (0.86 \pm 0.19) = (4.7 \pm 1.0) \times 10^{-3}. \quad (18)$$

Thus, the two-loop amplitude is more than two orders of magnitude larger than the leading logarithmic result in eqn. (13) and four orders of magnitude larger than the one-loop contribution in eqn. (2). The three contributions to the coefficient A are summarized in table 1.

One-loop electroweak diagrams	$ \Delta A_{1\ loop} \sim 2 \times 10^{-7}$
Leading logarithmic approximation	$ \Delta A_{LLA} = (8 \pm 3) \times 10^{-6}$
Dominant two-loop diagrams	$ A = (4.7 \pm 1.0) \times 10^{-3}$

Table 1. Summary of the contributions to the coefficient A .

The form of eqn. (14) ensures us that no further enhancement of the perturbative amplitude is expected at even higher orders. Any $\Delta S = 0$ charm decay must be Cabibbo-suppressed. The suppression by m_s/m_c must also remain. The latter suppression is actually rather mild, as one can see from eqns. (19) and (20) below. We have thus exhausted the possibility of finding large contributions by considering higher orders in perturbation theory.

4. The dominant two-loop contribution to the $c \rightarrow u\gamma$ amplitude is suppressed by $(m_s/m_c)^2$. Since m_s is smaller (though not by much) than the $\bar{\Lambda}$ parameter of HQET, the nonperturbative contribution to a D-hadron decay may be equally (or more) important than the $c \rightarrow u\gamma$ contribution.

Some part of the nonperturbative contribution may be taken into account by replacing current quark masses in the r.h.s. of eqn. (14) by constituent ones. Since the ratio of constituent masses is closer to unity than the ratio of current masses, one might worry that nonperturbative contributions may tend to bring back the GIM cancellations between s and d quarks. It is easy to explicitly check that this is not the case. For the constituent masses $m_d^{con} = 0.3$ GeV and $m_s^{con} = 0.45$ GeV one obtains

$$f[(m_s^{con}/m_c)^2] - f[(m_d^{con}/m_c)^2] \simeq -0.68 - 0.48 i, \quad (19)$$

while for the current masses one has

$$f[(m_s/m_c)^2] - f[(m_d/m_c)^2] \simeq -0.24 - 0.68 i. \quad (20)$$

We see that the resulting amplitude does not decrease (it is even larger) and conclude that the enhancement effect is rather robust.

Ignoring the nonperturbative terms and normalizing the $c \rightarrow u\gamma$ rate to the semileptonic decay rate (analogously to what one usually does in the $b \rightarrow s\gamma$ case), we would obtain the following $c \rightarrow u\gamma$ contribution to the branching ratio of $\Delta S = 0$ weak radiative D-meson decay:

$$\Delta BR[D \xrightarrow{\Delta S=0} X\gamma]_{c \rightarrow u\gamma} = \frac{\alpha_{QED}}{\pi} \frac{6|A|^2}{|V_{cd}|^2} BR[D \xrightarrow{\Delta S=0} X\bar{e}\nu]. \quad (21)$$

This amounts to roughly 5×10^{-8} for D^+ and to 2.5×10^{-8} for D^0 . Although it is dramatically increased over the previous results, it remains very small. Compared to $c \rightarrow u\bar{d}\bar{d}\gamma$ and $c \rightarrow us\bar{s}\gamma$, the $c \rightarrow u\gamma$ channel is suppressed (in branching ratio) by a factor of order $|A/V_{cd}|^2 \sim 5 \times 10^{-4}$. We conclude that there is little chance to extract the $c \rightarrow u\gamma$ contribution even with use of the photon energy spectra.

The obtained branching ratio is roughly confirmed when one looks at the exclusive decays $D \rightarrow \rho\gamma$. Assuming its BR to be down by about a factor of ten from the inclusive one, the contribution of $c \rightarrow u\gamma$ amounts to an exclusive BR of about 5×10^{-9} while the four-Fermi processes would give 10^{-5} . This is in satisfactory agreement with a recent sum rule calculation [9] of $D \rightarrow \rho\gamma$.

Since the $c \rightarrow u\gamma$ contribution in eqn. (21) turns out to be suppressed by both $(\alpha_s(m_c)/\pi)^2$ and powers of m_s/m_c , it is by no means the largest even among subdominant contributions. Decay modes with more gluons may have larger BR . For instance, the $c \rightarrow u\gamma$ *gluon* contribution to $BR[D \xrightarrow{\Delta S=0} X\gamma]$ is proportional only to linear $\alpha_s(m_c)/\pi$. It can be written in the same form as eqn. (21) but with $|A|$ replaced by 1.1×10^{-2} . Consequently, it is a factor of 5 larger in the rate than the $c \rightarrow u\gamma$ contribution. The quoted numerical value of the $c \rightarrow u\gamma$ *gluon* contribution has been obtained with use of analogous $b \rightarrow s\gamma$ *gluon* results from ref. [10] where appropriate replacements of electric charges had to be made.

5. To conclude, we have pointed out an interesting enhancement of the (formally) higher order contributions in the $c \rightarrow u\gamma$ decay. Nevertheless, the corresponding contribution to the inclusive weak radiative charmed hadron decay most probably remains screened by other channels.

6. Note added: The present version of this paper differs from the original one by correcting a mistake in the negligible one-loop contribution in eqn. (2). It was noticed by Q. Ho-Kim

and X.Y. Pham in hep-ph/9906235.

References

- [1] J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B247 (1990) 399.
- [2] M. Neubert, Phys. Rept. 245 (1994) 259 and references therein.
- [3] G. Burdman, E. Golowich, J. Hewett and S. Pakvasa, Phys. Rev. D52 (1995) 6383.
- [4] B. Bajc, S. Fajfer and R.J. Oakes, preprint IJS-TP-95/13 (hep-ph/9509361)
- [5] A. J. Buras, M. Misiak, M. Münz, and S. Pokorski, Nucl. Phys. B424 (1994) 374.
- [6] F.J. Gilman and M.B. Wise Phys. Rev. D20 (1979) 2392.
- [7] C. Greub, T. Hurth and D. Wyler, preprint SLAC-PUB-7113 (hep-ph/9602281); preprint SLAC-PUB-7144 (hep-ph/9603404).
- [8] M. Jamin and M. Münz, Z. Phys. C66 (1995) 633.
- [9] A. Khodjamirian, G. Stoll and D. Wyler, Phys. Lett. B358 (1995) 129.
- [10] A. Ali and C. Greub, Z. Phys. C49 (1991) 431; Phys. Lett. B259 (1991) 182.