Charm Radiative Decays with Neutral Mesons $D^0 \to \bar{K}^0 \pi^0 \gamma$, $D^0 \to \bar{K}^0 \eta(\eta') \gamma$

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ABSTRACT

The radiative decays $D^0 \to \bar{K}^0 P^0 \gamma$ with nonresonant $\bar{K}^0 P^0$ ($P^0 = \pi^0, \eta, \eta'$) are considered within the framework which combines heavy quark effective theory and the chiral Lagrangian. Due to neutral mesons the amplitudes do not have bremsstrahlung contributions. We assume factorization for the weak matrix elements. Light (virtual) vector mesons are found to give the main contribution to the decay amplitudes. The decay $D^0 \to \bar{K}^0 \pi^0 \gamma$ is predicted to have a branching ratio of 3×10^{-4}, with comparable contributions from parity-conserving and parity-violating parts of the amplitude. The decays with $\eta(\eta')$ in the final state are expected with branching ratios of 1.1×10^{-5} and 0.4×10^{-7} respectively and are mainly parity conserving.

Radiative weak decays of mesons have proved to be a fertile ground for the investigation of models of the strong interactions involved, and of the nonleptonic weak Hamiltonian. This is true for long-distance dominated K decays [1] as well as for B radiative decays, driven by short-distance dynamics [2-4]. One is lead to expect that likewise, radiative decays of D mesons [5-10] will bring similar useful insights, as well as providing possible checks [11-15] for physics beyond the standard model.

In a recent paper [16], we have initiated the study of radiative decays of type $D \to K\pi\gamma$, with non resonant $K\pi$. In [16] we studied the Cabibbo allowed decays which have a bremsstrahlung component, $D^+ \to \bar{K}^0\pi^+\gamma$ and $D^0 \to K^-\pi^+\gamma$. The theoretical framework used is that of the effective Lagrangian which contains both heavy flavor symmetry and $SU(3)_L \times SU(3)_R$ chiral symmetry [17] and the calculation was carried out with the factorization approximation for the weak matrix elements. The merging of this framework with the known QED requirements for the bremsstrahlung radiation necessitated the development of a specific technique for the treatment [16] of these decays.

In the present article we undertake the study of decays with neutral mesons $D^0 \to \bar{K}^0\pi^0(\eta,\eta')\gamma$, which are likewise Cabibbo allowed but do not involve a bremsstrahlung component. Due to this feature, these purely direct radiative decays may provide a cleaner testing ground for the study of the dynamics involved and for exploring the suitability of the theoretical framework employed in their calculation. In [16] it was found that the branching ratio for the parity-conserving part of the decay $D^0 \to K^-\pi^+\gamma$, which part is of purely "direct" nature, can be as high as 10^{-4}, while the total branching ratio for photon energies above 50 MeV is close to 10^{-3}. Such relatively high rates, especially for the direct transition, strongly motivates the desirability of studying the decays with neutral mesons which we consider here.

In decays of this type, one does not expect any significant short-distance contribution [5-10]. Since the decays $D^0 \to \bar{K}^0P^0\gamma$ involve transitions between a rather heavy meson and pseudoscalars, we adopt for our calculation the heavy quark chiral Lagrangian ($HQ\chi L$) containing heavy flavor and $SU(3)_L \times SU(3)_R$ symmetries as the appropriate theoretical framework. However, since virtual vector mesons may play an important role in these decays in the ($P\gamma$) channels, we need to complement the Lagrangian [17] with the light vector mesons. For this purpose we use the generalization of $HQ\chi L$ by Casalbuoni et al. [18] to include the vector mesons in the Lagrangian, in which the original symmetry is now broken spontaneously to diagonal $SU(3)_V$ [19]. This framework is described in detail in Refs. [8, 18], henceforth we recapitulate only the main features. The light meson part of the Lagrangian is written as

$$\begin{aligned}\mathcal{L}_{light} &= -\frac{f^2}{2}\{tr(\mathcal{A}_\mu\mathcal{A}^\mu) + a\,tr[(\mathcal{V}_\mu - \hat{\rho}_\mu)^2]\} \\ &+ \frac{1}{2g_v^2}tr[F_{\mu\nu}(\hat{\rho})F^{\mu\nu}(\hat{\rho})]\ . \end{aligned} \tag{1}$$

with $a = 2$ for exact vector meson dominance [19]. The vector and axial-vector currents are given by

$$\mathcal{V}_\mu = \frac{1}{2}(u^\dagger D_\mu u + u D_\mu u^\dagger) \qquad \mathcal{A}_\mu = \frac{1}{2}(u^\dagger D_\mu u - u D_\mu u^\dagger)\ , \tag{2}$$

where $u = \exp\left(\frac{i\Pi}{f}\right)$, Π being the matrix of the pseudoscalar fields and $f = f_\pi = 132$ MeV. D_μ is the covariant derivative. Moreover, $\hat{\rho}_\mu = i\frac{g_v}{\sqrt{2}}\rho_\mu$, with ρ_μ being the matrix of the vector fields:

$$\rho_\mu = \begin{pmatrix} \frac{\rho^0_\mu+\omega_\mu}{\sqrt{2}} & \rho^+_\mu & K^{*+}_\mu \\ \rho^-_\mu & \frac{-\rho^0_\mu+\omega_\mu}{\sqrt{2}} & K^{*0}_\mu \\ K^{*-}_\mu & \bar{K}^{*0}_\mu & \Phi_\mu \end{pmatrix}. \tag{3}$$

For the vector coupling $g_v(m_V^2)$ we use experimentally determined values from leptonic decays [20] of vector mesons, which accounts for the symmetry breaking effects.

In the heavy sector, to order $O(p)$ one has the Lagrangian

$$\mathcal{L}_{str} \;=\; Tr(H_a iv\cdot D_{ab}\bar{H}_b) + igTr(\bar{H}_a H_b \gamma_\mu A^\mu_{ba}\gamma_5), \tag{4}$$

where $D^\mu_{ab}H_b = \partial^\mu H_a - H_b V^\mu_{ba}$, while the trace Tr runs over Dirac indexes. The flavor indexes are denoted as a and b. Both the heavy pseudoscalar and the heavy vector meson are incorporated in a 4x4 matrix

$$H_a = \frac{1}{2}(1+\not{v})(P_{\mu a}\gamma^\mu - P_{5a}\gamma_5) \tag{5}$$

The strong coupling constant has been determined [21] recently from $D^* \to D\pi$ decay to be $g \approx 0.59$ and v_μ is the four-velocity of D meson.

The electromagnetic interaction is introduced by gauging the Lagrangians (1) and (4) with the U(1) photon field, thus amending appropriately the covariant derivatives. However, the gauging procedure alone does not generate the $D^* \to D\gamma$ transition, thus requiring the introduction of additional terms $\mathcal{L}_c$. There are two such terms in the framework used, giving the direct photon-heavy quark interaction with strength λ' (being of the order $1/m_Q$) and a light vector meson-heavy meson interaction with strength λ (being of the order $1/\lambda_\chi$ where λ_χ is the chiral perturbation theory scale). The second term provides photon emission via vector meson dominance (VMD). Thus,

$$\mathcal{L}_c \;=\; -\lambda' Tr[H_a\sigma_{\mu\nu}F^{\mu\nu}(B)\bar{H}_a] + i\lambda Tr[H_a\sigma_{\mu\nu}F^{\mu\nu}(\hat{\rho})_{ab}\bar{H}_b]. \tag{6}$$

An additional contributing term to the radiative decays via VMD is the Wess-Zumino-Witten anomalous interaction for the light sector, given by [22]:

$$\mathcal{L}^{(1)}_{odd} \;=\; -4\frac{C_{VV\Pi}}{f}\epsilon^{\mu\nu\alpha\beta}Tr(\partial_\mu\rho_\nu\partial_\alpha\rho_\beta\Pi). \tag{7}$$

In the calculation, instead of using the SU(3) symmetric coupling, we shall rely on the experimentally measured effective couplings of the $V \to P\gamma$ transitions [23]. For λ, λ' we take $\lambda = -0.49\ GeV^{-1}$, $\lambda' = -0.102\ GeV^{-1}$, as determined from $D^{*+,0}$ electromagnetic and strong decays [16], the signs chosen as to conform with the quark model. The effective weak $\Delta c = 1$ nonleptonic Lagrangian of relevance to the decays investigated here is

$$\mathcal{L}_{NL}^{eff}(\Delta c = \Delta s = 1) \quad = \quad -\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* [a_1 O_1 + a_2 O_2], \tag{8}$$

where $O_1 = (\bar{s}c)^{\mu}_{V-A}(\bar{u}d)_{\mu,V-A}$, $O_2 = (\bar{u}c)^{\mu}_{V-A}(\bar{s}d)_{\mu,V-A}$ and V_{ij} are the CKM matrix elements. The effective Wilson coefficients are taken as $a_1 = 1.26$, $a_2 = -0.55$ [24]. In our calculation we rely on factorization, which implies $< P^0 \bar{K}^0 | O_1 | D^0 >= 0$. The heavy-light weak current is bosonized as [17, 18]

$$\begin{aligned} J_{Q_a}^{\mu} = \ & \tfrac{1}{2} \quad i\alpha Tr[\gamma^{\mu}(1-\gamma_5) H_b u_{ba}^{\dagger}] \\ & + \quad \alpha_1 Tr[\gamma_5 H_b (\hat{\rho}^{\mu} - \mathcal{V}^{\mu})_{bc} u_{ca}^{\dagger}] \\ & + \quad \alpha_2 Tr[\gamma^{\mu}\gamma_5 H_b v_{\alpha} (\hat{\rho}^{\alpha} - \mathcal{V}^{\alpha})_{bc} u_{ca}^{\dagger}] + \ldots , \end{aligned} \tag{9}$$

The constant α is then related to the decay constant f_D by $\alpha = f_D\sqrt{m_D}$, while α_1, α_2 are determined from the values of the form factors appearing in $D \to V l \nu$ decays, as explained in [16]. Their numerical values are then $|\alpha_1| = 0.156$ GeV$^{1/2}$, $|\alpha_2| = 0.052$ GeV$^{1/2}$.

The general Lorentz decomposition of the $D^0 \to \bar{K}^0 P^0 \gamma$ amplitude is given by

$$\mathcal{M} = -\frac{G_F}{\sqrt{2}} V_{du} V_{cs}^* \Big(F_1 \left((q\cdot\varepsilon)(p\cdot k) - (p\cdot\varepsilon)(q\cdot k) \right) + F_2 \epsilon^{\mu\alpha\beta\gamma} \varepsilon_{\mu} v_{\alpha} k_{\beta} q_{\gamma} \Big) \ , \tag{10}$$

where F_1, F_2 are the electric and magnetic transitions, which are respectively parity-violating and parity-conserving. The four-momenta of P^0, K and the photon are q, p, k respectively and ε_{μ} is the polarization vector of the photon.

In Fig.1 we exhibit the Feynman diagrams contributing to the $D^0 \to \bar{K}^0 \pi^0 \gamma$ decay. The diagrams denoted A_i (parity-violating) derive from the $< \pi^0 | J | D^0 >< \bar{K}^0 | J | 0 >$ term; the same term gives the parity-conserving terms B_i, while the parity-conserving terms D_i come from $< 0 | J | D^0 >< \pi^0 \bar{K}^0 | J | 0 >$. The amplitudes A_i, B_i, D_i are related to the F_1, F_2 form factors by

$$F_1(D^0 \to \bar{K}^0 P^0 \gamma) = A_1 \ , \tag{11}$$

$$F_2(D^0 \to \bar{K}^0 P^0 \gamma) = \sum_{i=1}^{3} B_i + \sum_{i=1}^{2} D_i \ . \tag{12}$$

The sums for each row of the amplitudes in Fig.1 are gauge invariant. The explicit expressions of A_i, B_i, D_i are listed below, for the $D^0 \to \bar{K}^0 \pi^0 \gamma$ decay.

$$A_1 = i\frac{\sqrt{2}}{2} e g \frac{f_D f_K}{f_\pi} \frac{v\cdot k}{v\cdot k + v\cdot q + \Delta} \left(\frac{1}{v\cdot k + \Delta} - \frac{1}{v\cdot q + \Delta} \right) \left(2\lambda' - \frac{\sqrt{2}}{2} \lambda g_v \left(\frac{q_\omega}{3m_\omega^2} + \frac{q_\rho}{m_\rho^2} \right) \right)$$

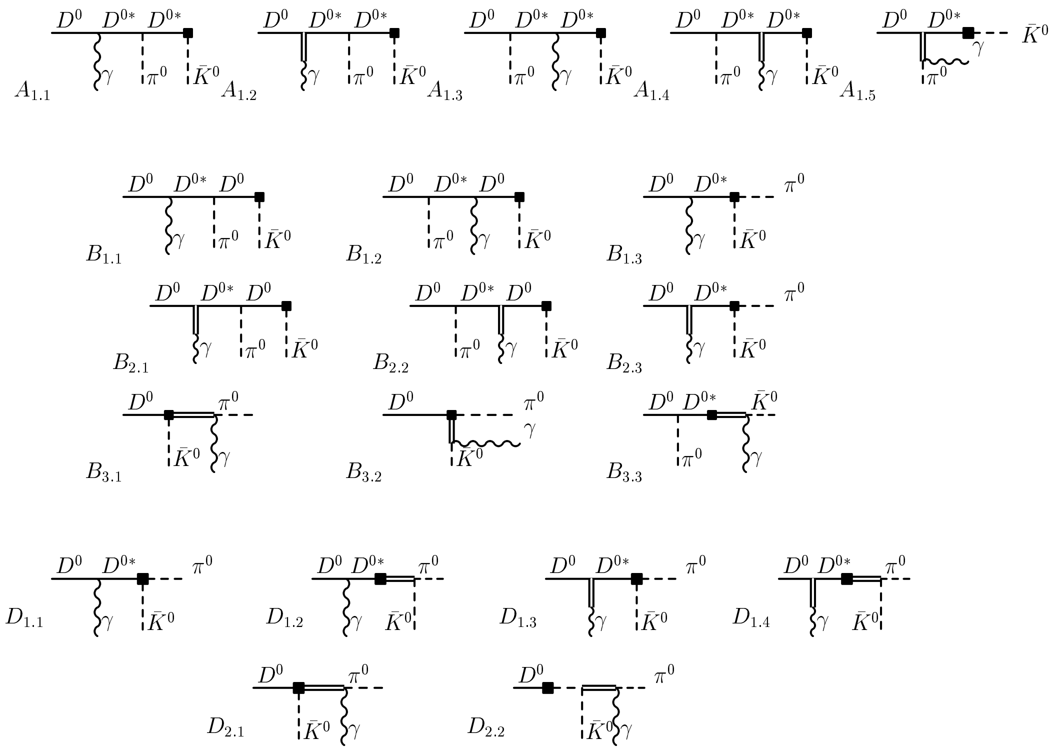

Figure 1: Feynman diagrams

$$
- ief_D f_K g_v \lambda M \frac{(v\cdot k)(p\cdot k)^2}{(v\cdot(q+k)+\Delta)} \left(\frac{g_{\rho\pi\gamma}}{(q+k)^2 - m_\rho^2 + i\Gamma_\rho m_\rho} + \frac{g_{\omega\pi\gamma}}{(q+k)^2 - m_\omega^2 + i\Gamma_\omega m_\omega} \right) , \tag{13}
$$

$$
\begin{aligned}
B_1 &= -\sqrt{2} eM \frac{f_D f_K}{f_\pi} \lambda' \left(\frac{1}{v\cdot k + \Delta} + g\frac{v\cdot p}{(v\cdot q + v\cdot k)(v\cdot k+\Delta)} + g\frac{v\cdot p}{(v\cdot q + v\cdot k)(v\cdot q+\Delta)} \right) , \\
B_2 &= \frac{1}{2} Me \frac{f_D f_K}{f_\pi} \lambda g_v \left(\frac{q_\omega}{3m_\omega^2} + \frac{q_\rho}{m_\rho^2} \right) \left(\frac{1}{v\cdot k + \Delta} + g\frac{v\cdot p}{(v\cdot q + v\cdot k)(v\cdot k+\Delta)} \right. \\
&\qquad \left. + g\frac{v\cdot p}{(v\cdot q + v\cdot k)(v\cdot q+\Delta)} \right) , \qquad (14) \\
B_3 &= \sqrt{M} eg_v f_K(\alpha_1 M - \alpha_2 v\cdot p) \left(\frac{g_{\rho\pi\gamma}}{(q+k)^2 - m_\rho^2 + i\Gamma_\rho m_\rho} + \frac{g_{\omega\pi\gamma}}{(q+k)^2 - m_\omega^2 + i\Gamma_\omega m_\omega} \right) \\
&\quad - \frac{\sqrt{2}}{2} e g_{\bar{K}^{0*}\bar{K}^0\gamma} g_{K^*} \frac{f_D}{f_\pi} M \frac{1 + g\frac{M - v\cdot q}{v\cdot q + \Delta}}{(k+p)^2 - m_{K^*}^2 + i\Gamma_{K^*} m_{K^*}} ,
\end{aligned}
$$

$$D_1 = \frac{\sqrt{2}}{2} M e f_D \frac{1}{v\cdot k + \Delta} \left(1 + \frac{m^2_{K^*}}{(p+q)^2 - m^2_{K^*} + i m_{K^*}\Gamma_{K^*}}\right)\left(2\lambda' - \frac{\sqrt{2}}{2}\lambda g_v \left(\frac{q_\omega}{3m^2_\omega} + \frac{q_\rho}{m^2_\rho}\right)\right) ,$$

$$D_2 = M\frac{\sqrt{2}}{2} e \frac{f_D}{f_\pi}\left(\frac{g_\rho g_{\rho\pi\gamma}}{(q+k)^2 - m^2_\rho + i m_\rho \Gamma_\rho} - \frac{g_\omega g_{\omega\pi\gamma}}{(q+k)^2 - m^2_\omega + i m_\omega\Gamma_\omega}\right) \tag{15}$$

$$-\frac{\sqrt{2}}{4} e f_D f_K \frac{M^3}{(M^2 - m^2_{\bar{K}^0})}\left(\frac{m^2_\rho}{g_\rho}\frac{g_{\rho\pi\gamma}}{(q+k)^2 - m^2_\rho + i m_\rho\Gamma_\rho} - \frac{m^2_\omega}{g_\omega}\frac{g_{\omega\pi\gamma}}{(q+k)^2 - m^2_\omega + i m_\omega \Gamma_\omega}\right) .$$

The treatment of the decays to η, η' requires the inclusion of $\eta - \eta'$ mixing. This is usually performed [25] by the mixing of singlet and octet states via a unitary matrix characterized by an angle θ. The analysis of various decays involving η, η' and of meson masses has led to a mixing angle in the range (-9°) to (-23°). However, a treatment [26] based on chiral perturbation theory and a detailed phenomenological analysis indicates that this simple scheme is inadequate and must be extended [26] to include two mixing angles θ_0 and θ_8, in addition to the two decay constants. Here we follow this approach with $|\eta >= \cos\theta_8|\eta_8 > - \sin\theta_0|\eta_0 >$ and $|\eta' >= \sin\theta_8|\eta_8 > + \cos\theta_0|\eta_0 >$. For the $D^0 \to \bar{K}^0\eta\gamma$ and $D^0 \to \bar{K}^0\eta'\gamma$ amplitudes the same form as Eqs. (13-15) holds, except for replacement of constants. In order to obtain these amplitudes one has to replace in above Eqs. f_π by $f_\eta/(K^\eta_d\sqrt{2})$ or $f_{\eta'}/(K^{\eta'}_d\sqrt{2})$, while in amplitude D_1 one replaces $\left(1 + \frac{m^2_{K^*}}{(p+q)^2 - m^2_{K^*} + i m_{K^*}\Gamma_{K^*}}\right)$ by $\sqrt{2}\left(K^\eta_d + (K^\eta_d - K^\eta_s)\frac{m^2_{K^*}}{(p+q)^2 - m^2_{K^*}}\right)$. Moreover, one replaces $g_{\rho\pi\gamma}$, $g_{\omega\pi\gamma}$ by $g_{\rho\eta(\eta')\gamma}$ and $g_{\omega\eta(\eta')\gamma}$. The factors K^η_d, K^η_s, $K^{\eta'}_d$, $K^{\eta'}_s$ are octet-singlet mixing factors, given by

$$K^\eta_d = \frac{\cos\theta_8}{\sqrt{6}} - \frac{\sin\theta_0}{\sqrt{3}} , \qquad K^{\eta'}_d = \frac{\sin\theta_8}{\sqrt{6}} + \frac{\cos\theta_0}{\sqrt{3}} , \tag{16}$$

$$K^\eta_s = -\frac{2\cos\theta_8}{\sqrt{6}} - \frac{\sin\theta_0}{\sqrt{3}} , \qquad K^{\eta'}_s = -\frac{2\sin\theta_8}{\sqrt{6}} + \frac{\cos\theta_0}{\sqrt{3}} . \tag{17}$$

A recent analysis by Feldmann, Kroll and Stech [28] has given further clarification of the theoretical basis of this scheme. Following their procedure [27, 28], we use $\theta_8 = -20.2^\circ$, $\theta_0 = -9.2^\circ$, $f_\eta = f_\pi$ and $f'_\eta = 1.13 f_\pi$. We have used these values in our calculation, as well as checking the sensitivity of the results to the scheme and to the values of above constants within an acceptable range.

Using now Eqs. (13-15) we calculate the differential decay distributions and total decay rates for $D^0 \to \bar{K}^0\pi^0\gamma$, and with appropriate replacements as indicated above, for $D^0 \to K^0\eta(\eta')\gamma$. Since ρ is a wide resonance we use a momentum dependent width

$$\Gamma_\rho(q^2) = \Gamma_\rho(m^2_\rho)\left(\frac{q^2 - 4m^2_\pi}{m^2_\rho - 4m^2_\pi}\right)^{3/2}\frac{m_\rho}{\sqrt{q^2}}\Theta(4m^2_\pi) . \tag{18}$$

We are interested in decays to nonresonant $K\pi$ final states and we also wish to delete from the final state resonant $(K\gamma)$ and $(P\gamma)$ configurations. Accordingly, we subtract the calculated rate given by the diagrams that include above configurations from the total rate. However, our results include the contributions of the remaining virtual vector mesons and their interference with the resonant ones. Our resulting prediction for the major radiative decay is

$$\mathrm{BR}(\mathrm{D}^0 \to \bar{\mathrm{K}}^0\pi^0\gamma)_{\mathrm{NR}} = 3.0 \times 10^{-4} \ . \tag{19}$$

The parity-conserving and the parity-violating parts of the amplitude contribute approximately 2/3 and 1/3 to the decay rate respectively. Now, if we do not subtract the direct vector mesons contribution, one gets a branching ratio of 3.0×10^{-3} for this channel. Obviously, the two cases have rather different Dalitz plots, the resonances being easily identified in it as they dominate the decay rate in the latter case. On the other hand, if we exclude vector mesons from our Lagrangian, the total decay rate is reduced to 2.6×10^{-8}. We have also calculated the size of the rate coming from the direct resonant process $D^0 \to \bar{K}^{*0}\gamma$ using our formalism and we find it to be more than two orders of magnitude smaller than (19), in general agreement with previous estimates [5, 8, 9].

For the decays to η, η', again after deleting resonant two body channels from the final state, we get:

$$\mathrm{BR}(\mathrm{D}^0 \to \bar{\mathrm{K}}^0\eta\gamma)_{\mathrm{NR}} = 1.1 \times 10^{-5} \ , \tag{20}$$

$$\mathrm{BR}(\mathrm{D}^0 \to \bar{\mathrm{K}}^0\eta'\gamma)_{\mathrm{NR}} = 4.3 \times 10^{-8} \ . \tag{21}$$

Allowing also $\eta\gamma$, $\eta'\gamma$ in the final state, coming from the intermediate $D^0 \to \rho(\omega)\gamma$ decay, these figures are raised to 3.9×10^{-5} and 1.4×10^{-7} respectively. Excluding completely the contribution of vector mesons, these branching ratios are lowered to 3.7×10^{-8} and 1.3×10^{-9}. In the decays to η, η', the dominant contribution to the decay rate is due to the parity conserving part of the amplitude.

The major role played by off-the-mass-shell vector mesons in these decays is evident from the above procedure. In particular, the ω-meson exchange is the dominant contribution to the pionic decay (Eq.(20)), while for the decay $D^0 \to \bar{K}^0\eta\gamma$, both ω and K^* exchange are giving major contributions. For the decay to η', the main contribution is due to K^*. The contribution of the ρ meson is generally much smaller, in all channels.

We have also found that the $D^0 \to K\eta\gamma$ rate is smaller than $D^0 \to K\pi^0\gamma$ rate partly due to phase space, but mostly due to the different values of the coupling constants involved. The smallness of the decay with η' in the final state is mainly due to phase space inhibition.

In Fig.2 we display the Dalitz plots for the three decay channels, and the photon spectra of these decays. The Dalitz plot coordinates are $m_{12} = \sqrt{(P-k)^2}$ on x axis and $m_{23} = \sqrt{(P-p)^2}$ on y axis and the spectrum is expressed in terms of $(m_{12})^2 = (M-k)^2$. In the main channel $D^0 \to \bar{K}^0\pi^0\gamma$ the peak of the parity-conserving contribution is at

$E_\gamma = 700$ MeV and of the parity-violating contribution at $E_\gamma = 500$ MeV. The total spectrum is predicted to peak around $E_\gamma = 650$ MeV. For the η decay, the spectrum peaks at $E_\gamma = 300$ MeV.

Before concluding we wish to make some observations concerning the appropriateness of the theoretical framework we used. By using the Lagrangians (4), (6)-(8) as well as the factorization approximation we can calculate also various decays of type $D \to VP$. In particular, we have calculated the rates for four channels of this type, which relate to the radiative decays we considered. Of these, ω channel is the most important one for the pionic decay while for the η channel, both the ω and K^* channels are giving the main contribution. We present now the results we obtained, giving in parentheses the observed branching ratios: $\mathrm{BR}(\mathrm{D}^0 \to \omega\bar{\mathrm{K}}^0) = 5.6\%(2.1\%)$, $\mathrm{BR}(\mathrm{D}^0 \to \rho\bar{\mathrm{K}}^0) = 7\%(1.2\%)$, $\mathrm{BR}(\mathrm{D}^0 \to \pi\mathrm{K}^*) = 11\%(3.2\%)$, $\mathrm{BR}(\mathrm{D}^0 \to \eta\mathrm{K}^*) = 3.2\%(1.9\%)$. The rates we obtain are within factor two-four (the relevant ones are on the lower side of this) of the experimental ones, which leads us to estimate that this is essentially the accuracy of our calculation, including also the uncertainty due to subtraction of the resonant channels from the total rate. We have checked the sensitivity of our results to the values of the mixing angles and the values of f_η, f'_η. Within reasonable values for these constants, the changes in rates are negligible.

We conclude by pointing out that the branching ratio of $D^0 \to \bar{K}^0\pi^0\gamma$ is quite large, due to the contribution of light virtual vector mesons. That makes it rather appealing for the experimental study since in this decay mode there is no need for special treatment of the bremsstrahlung component.

With the abundance of D's produced at B factories, at Tevatron and at the forthcoming charm factories, we look forward to experimental results for these decays, especially for the π and η modes, which are predicted to have sizable rates of 3×10^{-4} and 1.1×10^{-5} respectively.

Acknowledgments

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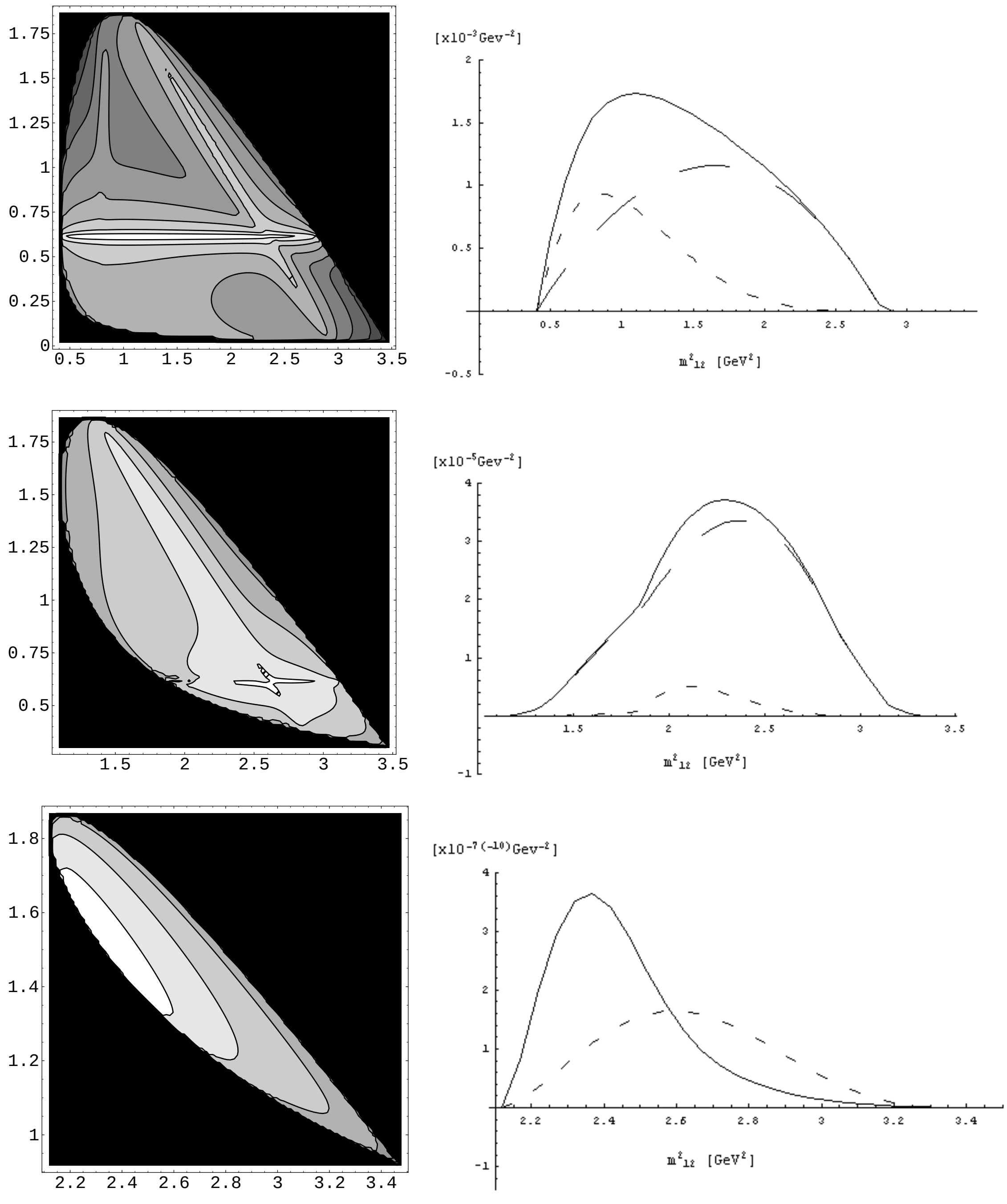

Figure 2: Dalitz plots (1. column) and photon spectra (2. column) for the $D^0 \to \bar{K}^0\pi^0\gamma$ decay (1. row), $D^0 \to \bar{K}^0\eta\gamma$ decay (2. row) and $D^0 \to \bar{K}^0\eta'\gamma$ decay (3. row). Short-dashed line: parity violating part. Long-dashed line: parity conserving part. Full-line: Total contribution. On the bottom right figure 10^{-7} label is for the full line and 10^{-10} for the dashed line.

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