

$c \rightarrow u\gamma$ in the minimal supersymmetric standard model

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ABSTRACT

The minimal supersymmetric standard model (MSSM) with universal soft breaking terms has a negligible effect on the rare $c \rightarrow u\gamma$ decay rate. We therefore study a general model with non-universal soft breaking terms constrained by vacuum stability and the experimental upper bound on the mass difference of the neutral charm mesons. It turns out that gluino exchange can enhance the standard model rate by up to two orders of magnitudes; in contrast, the contributions from charginos and neutralinos remain at least an order of magnitude below the QCD corrected standard model rate.

1. Introduction

Flavour changing neutral currents (FCNC) occur in the standard model only at the loop level where they are suppressed by the GIM mechanism. The corresponding processes are rare and suitable as probes for new physics, in particular for the popular low energy supersymmetry (SUSY) which has various new sources of flavour violation. Studying the FCNC processes may reveal the flavour structure of the soft supersymmetry breaking terms and shed light on the mechanism of SUSY breaking. There is a renewed interest in this issue, because models with effective supergravities [1] generally do not lead to universal soft breaking terms and tend to have large FCNC rates.

In this note we explore the sensitivity of the $c \rightarrow u\gamma$ decay rate in the framework of the minimal supersymmetric standard model (MSSM) with unbroken R-parity and arbitrary non-universal soft breaking terms. This process was considered before in ref. [2]; however,

only gluino exchanges were included and the constraints from vacuum stability were not applied. In addition, we take into account the QCD corrections within the standard model, which are seizable, and discuss the QCD corrections in the MSSM. Motivated by recent proposals for probing $c \rightarrow u\gamma$ in specific hadronic observables [3, 4, 5, 6], which are largely free of the 'uninteresting' long distance contributions, we analyze this decay in more detail.

In the standard model, the $c \rightarrow u\gamma$ amplitude is proportional to $\sum_{q=d,s,b} V_{cq}^* V_{uq} m_q^2$ at the one-loop level and the corresponding branching ratio $\sim 10^{-17}$ is negligibly small as the GIM mechanism is extremely effective in charm decays (the intermediate d and s quarks are very light compared to the W). The most important QCD corrections arise from the diagrams shown in Fig. 1. They drastically lift the GIM suppression; instead of power-like it becomes logarithmic [7]. The effective local interaction can be written as [7]

$$\begin{aligned} \mathcal{L}_{eff} &= -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{us} [c_7 \mathcal{O}_7 + c'_7 \mathcal{O}'_7] \\ \mathcal{O}_7 &= \frac{e}{16\pi^2} m_c \bar{u} \sigma_{\mu\nu} \frac{1}{2} (1 + \gamma_5) c F^{\mu\nu}, \quad \mathcal{O}'_7 = \frac{e}{16\pi^2} m_c \bar{u} \sigma_{\mu\nu} \frac{1}{2} (1 - \gamma_5) c F^{\mu\nu} \\ (c_7)_{\alpha_s}^{SM} &\simeq \frac{\alpha_s(m_c)}{4\pi} c_2(m_c) \{f[m_s^2/m_c^2] - f[m_d^2/m_c^2]\} \simeq -(0.007 + 0.020 i)[1 \pm 0.2] \\ (c'_7)_{\alpha_s}^{SM} &\simeq 0 \end{aligned} \tag{1}$$

for $m_c = 1.5$ GeV and $\alpha_s(m_c) = 0.31$, with functions f and c_2 given in [7]. The corresponding branching ratio in the standard model

$$Br_{\alpha_s}^{SM}(c \rightarrow u\gamma) = 6 \left| \frac{e V_{cs}^* V_{us}}{2\pi} \right|^2 \{ |c_7|^2 + |c'_7|^2 \} \frac{G_F^2 m_c^5}{192\pi^3 \Gamma(D^0)} \simeq 3 \times 10^{-8} \tag{2}$$

is still small and potentially sensitive to various extensions beyond the standard model.

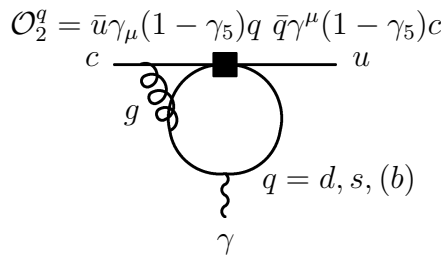


Figure 1: The most important QCD corrections to $c \rightarrow u\gamma$ are induced by the four-quark operator \mathcal{O}_2 at the two loop level. Only one of the representative diagrams in the standard model is shown.

The main generic problem for probing the $c \rightarrow u\gamma$ transition in hadron decays is that the most straightforward observables such as the decay rate $D \rightarrow \rho\gamma$ are largely dominated by long distance contributions and are independent of new physics [8]. However, there are observables which are dominated by the interesting short distance physics. For instance, the disturbing long distance contributions largely cancel in the difference of the decay rates $Br(D^0 \rightarrow \rho^0\gamma) - Br(D^0 \rightarrow \omega\gamma)$ as shown in [3]. Other possibilities to look for $c \rightarrow u\gamma$ are the $B_c \rightarrow B_u^* \gamma$ decay [4] and the violation of the equalities $Br(D_s^+ \rightarrow K^{*+}\gamma)/Br(D_s^+ \rightarrow$

$\rho^+\gamma) = \tan\theta_C$ [5], $Br(D^0 \rightarrow \rho^0\gamma)/Br(D^0 \rightarrow \bar{K}^{*0}\gamma) = \tan\theta_C/2$ [2, 6]. Similarly, the isospin violating quantity $\Gamma(D^+ \rightarrow \rho^+\gamma)/2\Gamma(D^0 \rightarrow \rho^0\gamma) - 1$ [9] might help to disentangle the different contributions. These observables are suitable for testing new scenarios that could significantly enhance the $c \rightarrow u\gamma$ rate, and are of particular interest in the context of SUSY flavour problem.

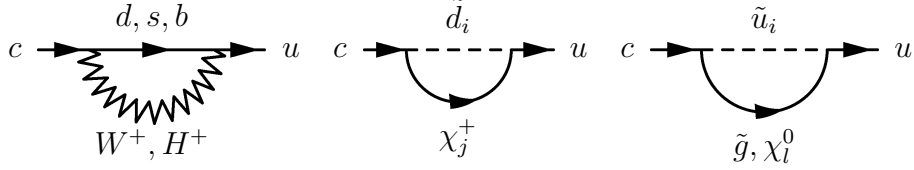


Figure 2: One-loop digrams for $c \rightarrow u\gamma$ decay in MSSM. Here $\tilde{q}_{i=1,\dots,6}$, $\chi_{j=1,2}^+$ and $\chi_{l=1,\dots,4}^0$ denote mass eigenstates for squarks, charginos and neutralinos. The photon is attached to any charged line.

2. The MSSM amplitudes

The one-loop diagrams for $c \rightarrow u\gamma$ transition in MSSM are shown in Fig. 2. The $\tilde{q}_{i=1,\dots,6}$, $\tilde{\chi}_{j=1,2}^+$ and $\tilde{\chi}_{l=1,\dots,4}^0$ denote mass eigenstates of squarks, charginos and neutralinos, respectively. We calculate the Willson coefficients at the high energy (m_W) scale for different contributions using the Feynman rules and notation presented in [10]

$$\begin{aligned}
(c_7)^{W^+} &= \frac{3}{2V_{cs}^*V_{us}} \sum_{q=d,s,b} V_{cq}^*V_{uq} \frac{m_q^2}{m_W^2} [e_d F_1\left(\frac{m_q^2}{m_W^2}\right) - F_2\left(\frac{m_q^2}{m_W^2}\right)] \quad (3) \\
(c_7)^{H^+} &= \frac{1}{2V_{cs}^*V_{us}} \sum_{q=d,s,b} V_{cq}^*V_{uq} \frac{m_q^2}{m_{H^+}^2} \left\{ tg^2\beta [e_d F_1\left(\frac{m_q^2}{m_{H^+}^2}\right) - F_2\left(\frac{m_q^2}{m_{H^+}^2}\right)] + [e_d F_3\left(\frac{m_q^2}{m_{H^+}^2}\right) - F_4\left(\frac{m_q^2}{m_{H^+}^2}\right)] \right\} \\
(c_7)^{\tilde{g}} &= -\frac{8}{3} \frac{\alpha_s}{\alpha_W V_{cs}^*V_{us}} e_u \sum_{i=1}^6 \frac{m_W^2}{m_{\tilde{u}_i}^2} [Z_U^{1i} Z_U^{2i*} F_2\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{u}_i}^2}\right) - \frac{m_{\tilde{g}}}{m_c} Z_U^{1i} Z_U^{5i*} F_4\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{u}_i}^2}\right)] \\
(c_7)^{\tilde{\chi}^+} &= -\frac{1}{V_{cs}^*V_{us}} \sum_{i=1}^6 \sum_{j=1}^2 \frac{m_W^2}{m_{\tilde{d}_i}^2} \left\{ a_{i1}^{j*} a_{i2}^j \left[-F_1\left(\frac{m_{\tilde{\chi}_j^+}^2}{m_{\tilde{d}_i}^2}\right) + e_d F_2\left(\frac{m_{\tilde{\chi}_j^+}^2}{m_{\tilde{d}_i}^2}\right) \right] \right. \\
&\quad \left. + \frac{m_{\tilde{\chi}_j^+}}{m_c} a_{i1}^{j*} b_{i2}^j \left[-F_3\left(\frac{m_{\tilde{\chi}_j^+}^2}{m_{\tilde{d}_i}^2}\right) + e_d F_4\left(\frac{m_{\tilde{\chi}_j^+}^2}{m_{\tilde{d}_i}^2}\right) \right] \right\} \\
(c_7)^{\tilde{\chi}^0} &= -\frac{1}{V_{cs}^*V_{us}} e_u \sum_{i=1}^6 \sum_{l=1}^4 \frac{m_W^2}{m_{\tilde{u}_i}^2} \left\{ A_{i1}^{l*} A_{i2}^l F_2\left(\frac{m_{\tilde{\chi}_l^0}^2}{m_{\tilde{u}_i}^2}\right) + \frac{m_{\tilde{\chi}_l^0}}{m_c} A_{i1}^{l*} B_{i2}^l F_4\left(\frac{m_{\tilde{\chi}_l^0}^2}{m_{\tilde{u}_i}^2}\right) \right\}
\end{aligned}$$

with $e_d = -1/3$, $e_u = 2/3$ and

$$\begin{aligned}
a_{iI}^j &= \frac{1}{g_2} \left[-\frac{e}{\sin \theta_W} Z_D^{Ii} Z_{1j}^- + \frac{\sqrt{2} m_d^I}{v_1} Z_D^{(I+3)i} Z_{2j}^- \right] V^{IJ*} \\
b_{iI}^j &= \frac{1}{g_2} \left[\frac{\sqrt{2} m_u^J}{v_2} Z_D^{Ii} Z_{2j}^{+*} \right] V^{IJ*} \\
A_{iI}^l &= \frac{1}{g_2} \left[-\frac{e}{\sqrt{2} \sin \theta_W \cos \theta_W} Z_U^{Ii*} \left(\frac{1}{3} Z_N^{1l} \sin \theta_W + Z_N^{2l} \cos \theta_W \right) - \frac{\sqrt{2} m_u^I}{v_2} Z_U^{(I+3)i*} Z_N^{4l} \right] \\
B_{iI}^l &= \frac{1}{g_2} \left[\frac{2\sqrt{2}e}{3 \cos \theta_W} Z_U^{(I+3)i*} Z_N^{1l*} - \frac{\sqrt{2} m_u^I}{v_2} Z_U^{Ii*} Z_N^{4l*} \right].
\end{aligned} \tag{4}$$

The expressions for c_7' are analogous, but they are multiplied by an overall factor m_u/m_c and m_c is replaced by m_u in (3). The matrices Z^\pm are the mixing matrices of chargino mass eigenstates $\chi_{j=1,2}^\pm$ with weak eigenstates $(\tilde{W}^+, \tilde{H}^+)$ and the matrix Z_N describes the mixing of the neutralino mass eigenstates $\chi_{l=1,\dots,4}^0$ with weak eigenstates $(\tilde{B}, \tilde{A}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ [10]. The functions $F_{1,\dots,4}$ are given in Appendix B of [11]. The matrices $Z_{U,D}$ diagonalize the squark mass matrices and depend on a large number of unknown soft breaking parameters in the general MSSM.

The number of these parameters entering the $c \rightarrow u\gamma$ amplitude is significantly reduced in the mass insertion approximation, where the expressions (3) are expanded to the first order in the physical squark mass splitting around the common average squark mass $m_{\tilde{q}}$ [12], i.e.

$$f\left(\frac{m_{\tilde{d}_i}^2}{m_{\tilde{\chi}_j^+}^2}\right) \simeq f\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{\chi}_j^+}^2}\right) + \frac{m_{\tilde{d}_i}^2 - m_{\tilde{q}}^2}{m_{\tilde{\chi}_j^+}^2} f'\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{\chi}_j^+}^2}\right). \tag{5}$$

This approximation works well [13] unless there are strong cancellations or large flavour violations (for instance a light sbottom with a mass around 4 GeV [14] could induce large flavor-violating effects. However, its contribution to $c \rightarrow u\gamma$ is negligible¹).

The summation over the intermediate squarks can be expressed in terms of the squark mass matrices $\mathcal{M}_{\tilde{U}}^2$ and $\mathcal{M}_{\tilde{D}}^2$ in the super-CKM basis [12]. In this basis for the squarks, the flavour structure of quark-squark-gaugino vertex is the same as the quark-quark-gauge boson vertex. We have

$$\sum_{i=1}^6 Z_U^{Ii} Z_U^{Ji*} = \sum_{i=1}^6 Z_D^{Ii*} Z_D^{Ji} = \delta^{IJ}, \quad \sum_{i=1}^6 Z_U^{Ii} Z_U^{Ji*} m_{\tilde{u}_i}^2 = (\mathcal{M}_{\tilde{U}}^2)^{IJ}, \quad \sum_{i=1}^6 Z_D^{Ii*} Z_D^{Ji} m_{\tilde{d}_i}^2 = (\mathcal{M}_{\tilde{D}}^2)^{IJ}. \tag{6}$$

In evaluating (3), we neglect the masses of the quarks except m_c because the magnetic transitions \mathcal{O}_7 and \mathcal{O}_7' (1) are of this order². The top quark mass m_t does not enter the

¹The sbottom-chargino exchange gives $(c_7)\chi^+ \propto V_{cb}^* V_{ub} Z_D^{1i*} Z_D^{2i} F(m_{\tilde{\chi}_+}^2/m_b^2) m_W^2/m_b^2 \propto V_{cb}^* V_{ub} Z_D^{1i*} Z_D^{2i} m_b^2 m_W^2/m_{\tilde{\chi}_+}^4$ (3) in the limit $m_b^2/m_{\tilde{\chi}_+}^2 \ll 1$, where the unitarity of Z_D has been applied.

²We neglect the terms proportional to m_u/m_c in c_7' .

amplitude and m_b comes only in combination with a small factor $V_{cb}^* V_{ub}$. This approximation is supported by the fact that $c_7^{W^+} \propto V_{cq}^* V_{uq} m_q^2$ (3) is about four orders of magnitude smaller than $(c_7)_{\alpha_s}^{SM}$ (1). Using the relations (6) and the explicit expressions for the squark mass matrices³ we get⁴

$$\begin{aligned}
(c_7)^{W^+}, (c_7)^{H^+}, (c_7')^{W^+}, (c_7')^{H^+}, (c_7')^{\tilde{\chi}^+} &\simeq 0 \\
(c_7)^{\tilde{g}} &= \frac{8 \alpha_s e_u}{3 \alpha_W V_{cs}^* V_{us}} \frac{m_W^2}{m_{\tilde{g}}^4} \left\{ -(M_{\tilde{U}}^2)_{LL}^{12} f_2\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) + (M_{\tilde{U}}^2)_{LR}^{12} \frac{m_{\tilde{g}}}{m_c} f_4\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) \right\} \\
(c_7)^{\tilde{\chi}^+} &= -\frac{1}{V_{cs}^* V_{us}} \sum_{j=1}^2 \overbrace{[V(M_{\tilde{D}}^2)_{LL} V^\dagger]^{12}}^{(M_{\tilde{U}}^2)_{LL}^{12}} \frac{m_W^2}{m_{\tilde{\chi}_j^+}^4} \left\{ |Z_{1j}^-|^2 \left[-f_1\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{\chi}_j^+}^2}\right) + e_d f_2\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{\chi}_j^+}^2}\right) \right] \right. \\
&\quad \left. - \frac{m_{\tilde{\chi}_j^+}}{\sqrt{2} m_W \sin \beta} Z_{1j}^- Z_{2j}^{+*} \left[-f_3\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{\chi}_j^+}^2}\right) + e_d f_4\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{\chi}_j^+}^2}\right) \right] \right\} \\
(c_7)^{\tilde{\chi}^0} &= -\frac{e_u}{V_{cs}^* V_{us}} \sum_{l=1}^4 \frac{m_W^2}{m_{\tilde{\chi}_l^0}^4} \left[\frac{1}{3} t g \theta_W Z_N^{1l*} + Z_N^{2l*} \right] \left((M_{\tilde{U}}^2)_{LR}^{12} \frac{m_{\tilde{\chi}_l^0}}{m_c} \left\{ -\frac{2}{3} t g \theta_W Z_N^{1j*} f_4\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{\chi}_l^0}^2}\right) \right\} \right. \\
&\quad \left. + (M_{\tilde{U}}^2)_{LL}^{12} \left\{ \frac{1}{2} \left[\frac{1}{3} t g \theta_W Z_N^{1l} + Z_N^{2l} \right] f_2\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{\chi}_l^0}^2}\right) + \frac{m_{\tilde{\chi}_l^0}}{2 m_W \sin \beta} Z_N^{4l*} f_4\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{\chi}_l^0}^2}\right) \right\} \right) \\
(c_7')^{\tilde{g}} &= \frac{8 \alpha_s e_u}{3 \alpha_W V_{cs}^* V_{us}} \frac{m_W^2}{m_{\tilde{g}}^4} (M_{\tilde{U}}^2)_{RL}^{12} \frac{m_{\tilde{g}}}{m_c} f_4\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right) \\
(c_7')^{\tilde{\chi}^0} &= -\frac{e_u}{V_{cs}^* V_{us}} \sum_{l=1}^4 \frac{m_W^2}{m_{\tilde{\chi}_l^0}^4} \left[\frac{1}{3} t g \theta_W Z_N^{1l} + Z_N^{2l} \right] (M_{\tilde{U}}^2)_{RL}^{12} \frac{m_{\tilde{\chi}_l^0}}{m_c} \left\{ -\frac{2}{3} t g \theta_W Z_N^{1j} f_4\left(\frac{m_{\tilde{q}}^2}{m_{\tilde{\chi}_l^0}^2}\right) \right\}
\end{aligned} \tag{7}$$

with

$$f_{1,\dots,4}(x) = \frac{\partial}{\partial x} \left[\frac{1}{x} F_{1,\dots,4}\left(\frac{1}{x}\right) \right]_{x=0} .$$

The relevant diagrams for the various contributions to c_7 (7) in the mass insertion approximation are shown in Fig. 3. The expression $V(M_{\tilde{D}}^2)_{LL} V^\dagger$ occurring in the chargino contributions (7) is equal to $(M_{\tilde{U}}^2)_{LL}$ at the unification scale [12]. At the weak scale, this relation is valid up to the corrections proportional to quark masses, which can be neglected within our approximation. The values of c_7' and the left-right induced c_7 are equal provided $|(M_{\tilde{U}}^2)_{LR}^{12}| \simeq |(M_{\tilde{U}}^2)_{RL}^{12}|$, which is indeed the case for the upper bounds given below (8).

QCD corrections will not drastically change our main result. As we will see, the contributions from charginos, neutralinos and left-left induced gluinos are generally smaller than those in the standard model. Thus the main effect found in ref. [7], namely that the corrections arising from \mathcal{O}_2 are dominant, remains true for them. We need to include the QCD corrections only for the left-right induced gluino contribution because it might give a larger

³A similar procedure is used in the Appendix of [12] for $b \rightarrow s\gamma$.

⁴We omit the terms proportional to $(M_{\tilde{U}}^2)_{RR}^{12}$ in c_7' , since the best enhancement in general MSSM is expected when the upper bound on Δm_D is saturated only by diagrams with $(M_{\tilde{U}}^2)_{LL}^{12}$ insertions (see Table 1), while $(M_{\tilde{U}}^2)_{RR}^{12}$ is set to zero.

branching ratio than the $\simeq 3 \times 10^{-8}$ of the standard model. Using Eqs. (37) and (45) of ref. [13], one finds that the anomalous dimensions of the relevant operators induce a 10% decrease of the amplitude (7), which is evaluated using $\alpha_s = \alpha_s(m_W) = 0.12$.

3. The constraints on the mass insertions

The Wilson coefficients (7) depend on the mass insertions $(M_{\tilde{U}}^2)_{LL}^{12}$, $(M_{\tilde{U}}^2)_{LR}^{12}$ and $(M_{\tilde{U}}^2)_{RL}^{12}$ in the super-CKM basis which incorporate the model dependence arising from the mechanism of supersymmetry breaking. In MSSM with universal soft breaking terms (i.e. models with gauge mediation [15]) the squark mass matrices are flavour diagonal at the unification scale. Flavour mixing is due to the renormalization group evolution down to the weak scale, because the Yukawa matrices for up and down squarks are not simultaneously diagonalizable. The off-diagonal elements in squark mass matrices at the weak scale are necessarily proportional to the product of Yukawa couplings, $(M_{\tilde{U}}^2)_{LL}^{12}$, $(M_{\tilde{U}}^2)_{LR}^{12} \propto \sum_{q=d,s,b} V_{cq}^* V_{uq} m_q^2$ [16], and the effect of this constrained MSSM scenario on $c \rightarrow u\gamma$ decay rate is negligible.

Supergravity scenarios [1] generally lead to the non-universal soft-breaking terms and FCNC can be used to probe their textures. The mass insertions are taken as free parameters in this case, with bounds coming from available experimental data and the consistency of the model. The mass insertions $(M_{\tilde{U}}^2)^{12}$ can be constrained by saturating the upper experimental upper limit on Δm_D , as done in [17]. Taking the recent CLEO result $(\Delta m_D \cos \delta + \Delta \Gamma_D \sin \delta/2)/(2\Gamma_D^2) < 0.04\%$ [18] and assuming the relative strong phase δ between $D^0 \rightarrow K^+\pi^-$ and $D^0 \rightarrow K^-\pi^+$ to be small, the upper bounds on mass insertions are given in Table 1.

$m_{\tilde{q}}^2/m_{\tilde{q}}^2$	$ (M_{\tilde{U}}^2)_{LL}^{12} /m_{\tilde{q}}^2$	$ (M_{\tilde{U}}^2)_{LR}^{12} /m_{\tilde{q}}^2$ $ (M_{\tilde{U}}^2)_{RL}^{12} /m_{\tilde{q}}^2$
0.3	0.03 $m_{\tilde{q}}/500$ GeV	0.04 $m_{\tilde{q}}/500$ GeV
1.0	0.06 $m_{\tilde{q}}/500$ GeV	0.02 $m_{\tilde{q}}/500$ GeV
4.0	0.14 $m_{\tilde{q}}/500$ GeV	0.02 $m_{\tilde{q}}/500$ GeV

Table 1: Limits on $(M_{\tilde{U}}^2)_{LL}^{12}$, $(M_{\tilde{U}}^2)_{LR}^{12}$ and $(M_{\tilde{U}}^2)_{RL}^{12}$, obtained by saturating the upper bound $\Delta m_D < 4.5 \times 10^{-14}$ GeV of CLEO [18] by gluino exchange [17].

Stricter upper bounds on $(M_{\tilde{U}}^2)_{LR}^{12}$ and $(M_{\tilde{U}}^2)_{RL}^{12}$ are obtained by requiring the minima of the scalar potential in the MSSM not to break electric charge or color and that they are bounded from below [19]. In particular

$$\frac{|(M_{\tilde{U}}^2)_{LR}^{12}|}{m_{\tilde{q}}^2}, \frac{|(M_{\tilde{U}}^2)_{RL}^{12}|}{m_{\tilde{q}}^2} \leq \sqrt{3} \frac{m_c}{m_{\tilde{q}}} \simeq 0.0023 \frac{500 \text{ GeV}}{m_{\tilde{q}}}. \quad (8)$$

The factor m_c in (8) cancels the factor m_c in $(M_{\tilde{U}}^2)_{LR}^{12}/m_c$ (7), which in turn comes about since the chirality flip in the squark state does not have to be accompanied by the chirality flip in the external c quark state.

Another upper bound [12]

$$(M_{\tilde{U}}^2)_{LL}^{12} \leq \max \left[(M_{\tilde{D}}^2)_{LL}^{12}, \frac{V^{12}}{V^{13}} (M_{\tilde{U},\tilde{D}}^2)_{LL}^{13}, \frac{V^{12}}{V^{32}} (M_{\tilde{U},\tilde{D}}^2)_{LL}^{32} \right] \quad (9)$$

is due to the relation $(M_{\tilde{U}}^2)_{LL} = V(M_{\tilde{D}}^2)_{LL}V^\dagger$. Although the constraint on $(M_{\tilde{D}}^2)_{LL}^{12}$ from Δm_K [17] is about 30% stronger than the constraint on $(M_{\tilde{D}}^2)_{LL}^{12}$, given in Table 1, the upper bound (9) is ineffective due to the large ratios V^{12}/V^{32} and V^{12}/V^{13} .

We have explicitly verified that the upper bounds on $(M_{\tilde{U}}^2)_{LL}^{12}$ (Table 1) and $(M_{\tilde{U}}^2)_{LR}^{12}$ (8) do not saturate the experimental value $(\Delta m_K)^{exp} \simeq 3.5 \times 10^{-15}$ GeV through chargino exchange⁵.

4. The results

We consider low-energy supersymmetry with masses of superpartners in the TeV range. We take $250 \text{ GeV} \leq m_{\tilde{q}}, m_{\tilde{g}} \leq 1000 \text{ GeV}$ with lower bounds given by the direct searches for these states [21]. The masses of charginos, neutralinos and the corresponding mixing matrices Z^\pm , Z_N can be expressed in terms of $tg\beta$, μ , m_1 and m_2 [10, 22]. We take $tg\beta$ and μ in the range $2.5 \leq tg\beta \leq 30$ and $100 \text{ GeV} \leq |\mu| \leq 300 \text{ GeV}$. We assume $m_1 \simeq m_2$ and determine this parameter by setting the mass of lighter chargino equal to the experimental lower bound $m_{\chi_1^\pm} > 90 \text{ GeV}$ [23] (we vary this parameter in the range 90 – 200 GeV). Points of parameter space, where the neutralino masses violate bounds from the direct experimental searches, are omitted. The technical problem of negative eigenvalues for chargino and neutralino mass matrices is solved following the discussion in Appendix A.3 of [22]. The value $\alpha_s = \alpha_s(m_W) = 0.12$ is used.

The predicted $c \rightarrow u\gamma$ branching ratios are given in Fig. 4 in a form of a scatter plot, with input parameters varying in ranges discussed above. The mass insertions $(M_{\tilde{U}}^2)_{LL}^{12}$ and $(M_{\tilde{U}}^2)_{LR,RL}^{12}$ are taken at the maximal values of the allowed ranges, given in Table 1 and Eq. (8), respectively. The QCD-corrected standard model prediction (1, 2) is represented by the dot on the left. The remaining dots represent branching ratios arising solely from the genuine supersymmetric contributions; as discussed above, QCD corrections hardly affect these (7) and are neglected. The rates that would result from the gluino, the chargino and the neutralino contributions (7) are shown separately. If $(M_{\tilde{U}}^2)_{LR}^{12}$ is close to the upper value given by the vacuum stability argument (8), the gluino exchange is dominated by the left-right mass insertion and can enhance the standard model rate (2) by two orders of magnitudes

$$Br(c \rightarrow u\gamma)_{LR} \simeq 6 \times 10^{-6} \quad (10)$$

for $m_{\tilde{q}} \sim m_{\tilde{g}} \sim 250 \text{ GeV}$. The process $c \rightarrow u\gamma$ is therefore particularly suitable for probing the universality of the trilinear soft-breaking A terms, that are responsible for the left-right mass insertions. If the dominant supersymmetric contribution comes from the $(M_{\tilde{U}}^2)_{LL}^{12}$ insertion (when $(M_{\tilde{U}}^2)_{LR}^{12}$ is negligible), then the gluino and chargino contributions are comparable

⁵The upper bound $(M_{\tilde{U}}^2)_{LR}^{12}$ (8) gives a Δm_K which is several orders of magnitudes below $(\Delta m_K)^{exp}$. The upper bounds $(M_{\tilde{U}}^2)_{LL}^{12}$ (Table 1) can render values close to, but not above $(\Delta m_K)^{exp}$, for low masses of the charginos. The expressions for the chargino contributions to Δm are presented in [20].

in size and in fact add up. In this case, the highest rate of genuine SUSY contribution is achieved at small $tg\beta$ and $\mu < 0$

$$Br(c \rightarrow u\gamma)_{LL} \simeq 10^{-9} \quad (11)$$

and would be screened by the standard model contribution (2).

5. Conclusions

We have examined the sensitivity of the $c \rightarrow u\gamma$ decay rate in the context of the minimal supersymmetric standard model. Not surprisingly, schemes with universal soft breaking terms have a negligible effect on this process. If the universality condition is relaxed, the chargino and neutralino contributions remain small and are at least one order of magnitude below the standard model rate. However, the gluino exchange can enhance the standard model rate of $Br(c \rightarrow u\gamma) \simeq 3 \times 10^{-8}$ by up to a factor of 200, given the various constraints on the relevant left-right mass insertions. This is quite a surprising result and makes this decay very attractive for flavor studies in supersymmetric models. To understand this drastic effect, we recall that the standard model branching ratio is essentially due to a two-loop diagrams in Fig. 1, because the one loop-contribution is heavily GIM suppressed. As a consequence, one may expect that the GIM non-suppressed two-loop amplitude is enhanced by a factor $\alpha_s(m_W)/(\alpha_W V_{cs}^* V_{us} \alpha_s(m_c))$ in a general MSSM, in qualitative agreement with our detailed numerical result.

Although unwanted long distance effects strongly affect the hadronic rates, it is possible to find observables where they largely cancel. An important observable is $R = [Br(D^0 \rightarrow \omega\gamma) - Br(D^0 \rightarrow \rho^0\gamma)]/Br(D^0 \rightarrow \omega\gamma)$ which can go up to $\mathcal{O}(1)$ in the general MSSM discussed here, as compared to a standard model prediction of $6 \pm 15\%$ [3]. Although the long distance effects are not so well under control [25], a value of R above 20% would be a clear signal for new physics. Present experimental upper bounds on the rare decays $D^0 \rightarrow \rho^0\gamma$ and $D^0 \rightarrow \rho^0\omega$ are at the level of 10^{-4} [24, 21], while theoretically predicted are at a level of 10^{-6} [8]. It is expected that BaBar, Belle, BTeV and possibly a tau-charm factory can observe these interesting decays.

Another observable which is relatively free of long distance effects is $Br(B_c \rightarrow B_u^*\gamma)$. While the standard model expectation is 1×10^{-8} [4], it could reach 1×10^{-6} in the MSSM. Such an enhancement can be observed at LHC, which is expected to produce 2×10^8 B_c mesons with $p_T(B_c) > 20$ GeV.

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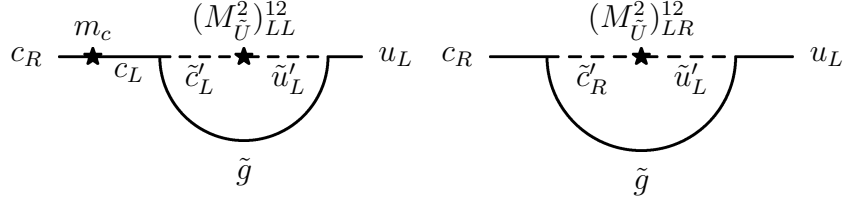
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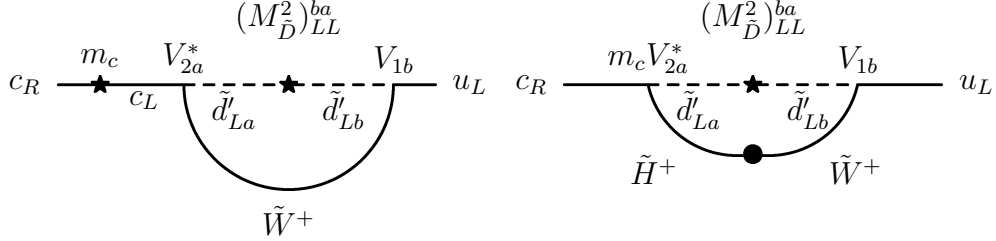
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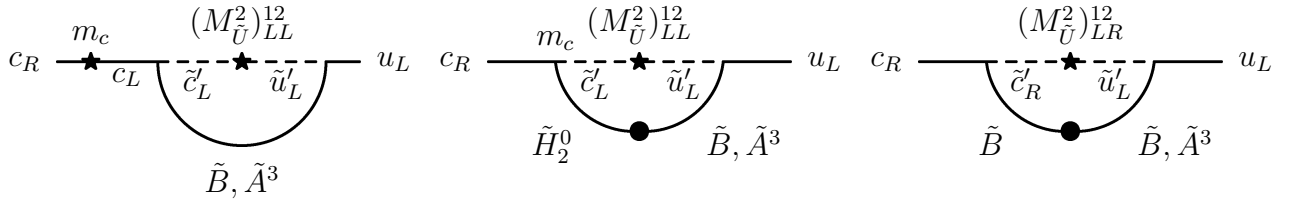
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(a) Dominant gluino exchange diagrams.



(b) Dominant chargino exchange diagrams with flavour index $a, b = 1, 2, 3$.



(c) Dominant neutralino exchange diagrams.

Figure 3: The one-loop diagrams for $c \rightarrow u\gamma$ decay in MSSM within the mass insertion approximation. Only the diagrams, that give raise to the local operator $\mathcal{O}_7 \propto \bar{u}_L \sigma^{\mu\nu} c_R$ (1), are shown. Quark masses are neglected and only the terms of the order of $\mathcal{O}(m_c)$ are kept. The stars denote mass insertions for quarks and squarks. The squark mass insertions $M_{\tilde{U}, \tilde{D}}^2$ and primed squark fields \tilde{q}' are represented in super-CKM basis. The dots denote mixing of different weak-isospin chargino or neutralino eigenstates. The photon is attached to any charged particle.

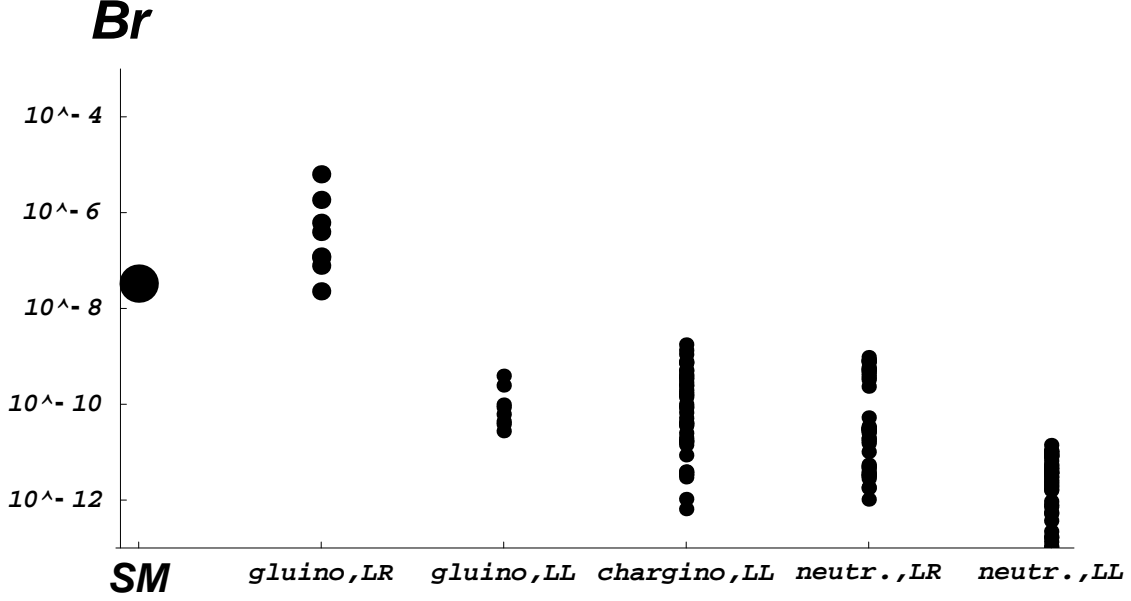


Figure 4: The predicted $c \rightarrow u\gamma$ branching ratios for various contributions in MSSM. The QCD-corrected standard model rate (1, 2) [7] is represented by the dot on the left. The remaining dots represent the rates arising solely from various supersymmetric contributions and do not include the QCD corrections (7): the gluino exchange (the contributions arising from $(M_{\tilde{U}}^2)_{LR,RL}^{12}$ and $(M_{\tilde{U}}^2)_{LL}^{12}$ are shown separately), the chargino exchange and the neutralino exchange (the contributions arising from $(M_{\tilde{U}}^2)_{LR,RL}^{12}$ and $(M_{\tilde{U}}^2)_{LL}^{12}$ are shown separately). The mass insertions $(M_{\tilde{U}}^2)_{LL}^{12}$ and $(M_{\tilde{U}}^2)_{LR,RL}^{12}$ are taken at the maximal values of the allowed ranges, given in Table 1 and Eq. (8), respectively. Other parameters are varying in the ranges, discussed in the text.