

Solutions for class #16 from Yosunism website

Yosunism website: <http://grephysics.yosunism.com>

Problem 74:

Thermodynamics \Rightarrow Entropy

Recall the definition of entropy to be $dS = dQ/T$. The heat is defined here as $dQ = mc dT$, and thus $S = \int mc dT/T$.

One is given two bodies of the same mass. One mass is at $T_1 = 500$ and the other is at $T_2 = 100$ before they're placed next to each other. When they're put next to each other, one has the net heat transferred being 0, thus $Q_1 = -Q_2 \Rightarrow T_f = (T_1 + T_2)/2 = 300$.

The entropy is thus, as in choice (B).

YOUR NOTES:

Problem 75:

Thermodynamics \Rightarrow Fourier's Law

Recall Fourier's Law $q = -k \nabla T$, where q is the heat flux vector (rate of heat flowing through a unit area) and T is the temperature and k is the thermal conductivity. (One can also derive it from dimensional analysis, knowing that the energy flux has dimensions of $J/(sm^2)$)

Fourier's Law implies the following simplification: $q = -k \frac{\Delta T}{\Delta l}$

The problem wants the ratio of heat flows $q_A/q_B = \frac{k_A l_B}{k_B l_A} = \frac{0.8 \times 2}{0.025 \times 4} = 32/2 = 16$, as in choice (D). (The problem gives $l_A = 4$, $l_B = 2$, and $k_A = 0.8$, $k_B = 0.025$.)

YOUR NOTES:

Problem 79:

Statistical Mechanics } Specific Heat

The specific heat at constant volume for high temperatures is $c_v = 7/2 R$. The specific heat at low temperatures is $3/2 R$. Why?

There are three contributions to the specific heat of a diatomic gas. There is the translational, vibrational, and rotational. At low temperatures, only the translational heat capacity contributes $U = 3/2 N k T \approx c_v T \rightarrow c_v = 3/2 Nk$. At high temperatures, all three components contribute, and one has $c_v = (3/2 + 1 + 1)Nk = 7/2 Nk$.

The general formula is

$$c_v = c_v(\text{translational}) + c_v(\text{rotational}) + c_v(\text{vibrational}) = Nk \left(\frac{3}{2} + 1 + \frac{2}{\exp(\hbar^2/(kT)) - 1} \right)$$

YOUR NOTES:

Problem 91:

Thermodynamics \Rightarrow } Second Law

The Second Law of thermodynamics has to do with entropy; that entropy can never decrease in the universe. One form of it states that *from hot to cold things flow*. A cooler body can thus never heat a hotter body. Since the oven is at a much lower temperature than the wanted sample temperature, the oven can only heat the sample to a maximum of 600K without violating the Second Law.

(This solution is due to David Latchman.)

(Also, since the exam is presumably written by theorists, one can narrow down the choices to either (D) or (E), since the typical theorist's stereotype of experimenters usually involves experimenters attempting to violate existing laws of physics---usually due to naivety.)

YOUR NOTES:

Problem 94:

Statistical Mechanics \Rightarrow } Internal Energy

The partition function is $Z = \sum e^{-\epsilon_i/kT} = 1 + e^{-\epsilon/kT}$. Internal energy is given by , as in choice (D).

YOUR NOTES:

Problem 58:

Optics \Rightarrow } Lensmaker Equations

Although this problem mentions lasers, no knowledge of quantum mechanics or even lasers is required. Instead, the problem can be solved as a simple geometric optics problem using the lensmaker's equation, $1/d_i + 1/d_o = 1/f$, relating the distances of the object, image, and the focus.

Since there are two convex lenses, one can treat the set-up as a telescope. The lens closest to the laser is the objective (with focus f_o) and the one closest to the bigger-radius well-collimated beam is the eyepiece f_e .

The laser-light comes in from $d_{o1} = \infty$, and thus one has $d_{i1} = f_o$, i.e., the image forms at the focal point.

Using the telescope equation, one has $M = f_o/f_e = 10 \Rightarrow f_e = f_o/10 = 15cm$, since one wants a final magnification of 10 (to wit: input beam is 1mm, output beam is 10mm). This narrows down the choices to just (D) and (E).

Since the distance between lenses for a telescope (with incoming light coming from infinity) is given by $d_{i1} + d_{o2} = f_o + f_e = 16.5$, which relates the distance of the image from the first lens to the distance of the object from the second lens, one arrives at choice (E). (Correction due to user tachyon.)

YOUR NOTES:

Problem 59:

Quantum Mechanics \Rightarrow } Laser

Not much understanding of lasers is required to solve this one; the basic idea of the problem tests the relation between photon energy and energy from the laser. Recalling the equation $E = hf = hc/\lambda \approx 12 E^{-7}/600 E^{-9} = 2 eV$, equates that to the energy (in eV) calculated from $P t = 10 E^3 \times 1 E^{-15} / 1.602 E^{-19}$. The answer comes out to choice (B).

YOUR NOTES:

Problem 60:

Wave Phenomena \Rightarrow } Light Doppler Shift

One can derive the Doppler Shift for light as follows:

For source/observer moving towards each other, one has the wavelength emitted from the source decreasing, thus $\lambda = (cdt - vdt) = (c-v)t_0\gamma$. Thus, $\lambda = (c-v)\gamma\lambda_0/c$.

For source/observer moving away from each other, one has the wavelength emitted from the source increasing, thus $\lambda = (cdt + vdt) = (c+v)t_0\gamma$. Thus, $\lambda = (c+v)\gamma\lambda_0/c$.

Where in the last equality in the above, one applies time dilation from special relativity, $t = t_0\gamma$ and the fact that $c = \lambda f = \lambda/t$ in general.

Now that one has the proper battle equipment, one can proceed with the problem.

This problem is essentially the difference in wavelengths seen from a red shift and blue shift, i.e., light moving towards and away from the observer.

, where the approximation $\gamma \approx 1$ is made since one assumes the particle is moving at a non-relativistic speed.

2 km is closest to choice (B).

YOUR NOTES:

Problem 82:

Optics \Rightarrow } Thin films

For a thin film of thickness t , one can easily find the condition for interference phenomenon. Since the light has to travel approximately $2t$ to get back to the original incidence interface, one has $2t = m\lambda$. However, since the light changes phase at the interface between air and glass (since glass has a higher index of refraction than air), the condition for constructive interference becomes $2t = m\lambda/2$, where $m \in \text{Odd}$.

One can create a table to determine the values of $t = m\lambda/4$.

m t

$$1 \quad \lambda/4 = 122\text{nm}$$

$$3 \quad 3\lambda/4 = 366\text{nm}$$

$$5 \quad 5\lambda/4 = 610\text{nm}$$

and so forth...

One thus finds that choice (E) is correct.

YOUR NOTES:

Problem 45:

Lab Methods \Rightarrow } High-pass filter

Recall the impedance formulae for capacitors $X_C = \frac{1}{\omega C}$ and inductors $X_L = \omega L$. The complex impedance is $Z = -iX_C + iX_L + R$, and the ac-version of Ohm's Law becomes: $V = IZ$.

For choice (E), one has $Z = R + iX_C \Rightarrow V_{in} = I(R + i(X_L - X_C)) \Rightarrow I = \frac{V_{in}(R + iX_C)}{R^2 + X_C^2}$, where in the last step, one multiplies top and bottom by the complex conjugate of the denominator impedance

Z. The voltage across the resistor is the voltage from ground, thus $V_{out} = IR = \frac{V_{in}(R + iX_C)R}{R^2 + X_C^2}$.

For high frequencies, one has $\omega \rightarrow \infty \Rightarrow X_C \rightarrow 0 \Rightarrow V_{out} = V_{in}R^2/R^2$.

For low frequencies, one has $\omega \rightarrow 0 \Rightarrow X_C \gg 1 \Rightarrow V_{out} = \frac{V_{in}R}{X_C} \rightarrow 0$.

Circuit (E) meets the given conditions.

(Incidentally, choice (D) is a low-pass filter giving $V_{out} \rightarrow 0$ for high frequencies.)

(For more on this, check out Horowitz' *The Art of Electronics*.)

YOUR NOTES:

Problem 72:

Lab Methods ⇒ } Negative Feedback

Negative feedback, according to Horowitz's *The Art of Electronics*, has to do with canceling out some of the input in the output. Although that might seem like redundantly adding noise to the system, it actually reduces the amplifier's gain, increases stability (by decreasing nonlinearity and distortion).

From that bit of info, two choices remain. Choice (A) and (B). Choose (A) because negating the feedback should not increase the amplitude.

YOUR NOTES: