Solutions for class #13 from Yosumism website

Yosumism website: http://grephysics.yosunism.com

Problem 17:

Quantum Mechanics⇒}Probability

The careless error here would be to just directly square the grids. When one remembers the significance of the meaning of the probability $P=\int|\langle\psi|\psi\rangle|^2 dV$, one finds that one must square the wave function, and not the grids.

The total probability is,

$\int_0^6 |\psi|^2 = 1+1+4+9+1+0 = 16$

The un-normalized probability from $x=2$to $x=4$is,

$\int_2^4 |\psi|^2 = 4+9 = 13$

The normalized probability is thus $13/16$, as in choice (E).

YOUR NOTES:

Problem 18:

Quantum Mechanics⇒}Probability

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YOUR NOTES:

Problem 33:

Quantum Mechanics⇒}Probability

Recall that $P(apple) = \int |\langle \psi_{apple} | \psi_{fruit} \rangle|^2 dx$

Given the wave function in terms of the spherical harmonic eigenfunctions, one has it totally easy. One has $\langle\psi| = (3Y_5^1 + 2Y_5^{-1})/\sqrt{38}$. Ketting the bra, one has, $P = (9+4)/38$, where one recalls the orthonormality of the spherical harmonic eigenfunctions. This is choice (C).

YOUR NOTES:

Problem 51:

Quantum Mechanics⇒}Infinite Well

The even wave functions always have nodes in the middle, and thus the probability density for states of even n vanish. (One can deduce this by fitting curves inside a box. The first state has no nodes in the middle, but a node at each end of the well. The second state has one node in the middle. The third state has two nodes, neither of which are in the middle. The fourth state has three nodes, one of which is in the middle.)

YOUR NOTES:

Problem 52:

Quantum Mechanics⇒}Spherical Harmonics

Y_l^m is the convention used for a spherical harmonic of eigenvalue m, l.
The only spherical harmonic proportional to $\sin\theta$ is $Y_1^{\pm 1}$. Recalling that the eigen-equation, $L_z\psi = m\hbar\psi$, one deduces that since $m = \pm 1$, the eigenvalues are $\pm\hbar$.

(Open call for a better solution: Is there a method that does not require memorizing the first few spherical harmonics?)

YOUR NOTES:

Problem 76:

Quantum Mechanics⇒}Uncertainty

One can make a good stab at this problem by applying the uncertainty principle.

I. If the average momentum of the packet is 0, then one violates the uncertainty principle. See IV.

II. Maybe.

III. Maybe.

IV. True, recall the *Gaussian* uncertainty principle $\Delta x \Delta k = \hbar/2$.

Since I is false, choices (A), (C), and (E) are out. Choices (B) and (D) remain. Take the conservative approach and choose (B).

YOUR NOTES:

Problem 77:

Quantum Mechanics⇒}Operators

This problem can be solved (without much knowledge of quantum mechanics) by noting the following general arithmetic trick: $ab=\frac{1}{2}\left((a+b)^2-a^2-b^2\right)$.

The problem gives the Hamiltonian $H=-JS_1\cdot S_2$, which has the same form as the arithmetic trick above. Thus, $H=-J\frac{1}{2}\left((S_1+S_2)^2-S_1^2-S_2^2\right)$.

Recalling some basic linear algebra, one can make use of the eigenvalue equations supplied with the problem defining the eigenvalues of the wanted operators, $S_i^2\psi_i=S_i(S_i+1)\psi_i$.

Thus, ,
where one has applied the eigenvalue equation above and generalized it for the case $(A+B^2)\psi=(A+B)(A+B+1)\psi$. From a bit of math manipulation, one has arrived at choice (D).

YOUR NOTES:

Problem 97:

Quantum Mechanics⇒}Probability Density Current

If one has forgotten the expression for the probability density current, then one needs not despair! If one remembers the vaguest definition of a ``probability current,'' one can solve this problem *without* the usage of the forgotten formula:

Recall that the probability current density J is the difference between incoming P(in) and outgoing P(out) probability densities. This is just the difference between the probability densities of a rightwards moving plane wave and a leftwards moving plane wave, since the probability density is related to the wave function by $P=|\psi|^2$

$J \propto P(in)-P(out)=|f(e^{-ikx})|^2-|f(e^{ikx})|^2$
The given wave function can be written in terms of plane waves,

$f(e^{ikx})$gives the coefficient for the leftwards (negative-direction) wave, while $f(e^{-ikx})$gives the coefficient for the rightwards traveling wave. The probability density for each wave is given by,

Plugging these quantities into the formula for the probability current density above (J), one gets,

$J \propto \frac{1}{2i}(\alpha\beta^*-\alpha^*\beta)$,which is choice (E).

Alternatively:

The formal expression for the probability current density can be effortlessly derived from recalling the definition of probability and Schrodinger's Equation---both of which *every* physics (or engineering) major should know by heart.

Probability is defined (in the Born Interpretation) as $P=\int|\Psi(x,t)|^2dx$. One should recall that $|A|^2=A^*A=AA^*$ in general (to wit: the absolute value squared of a complex expression is itself times its complex conjugate).

The time-dependent Schrodinger's Equation is

$\hbar i\frac{\partial\Psi}{\partial t}=H\Psi=-\frac{\hbar^2}{2m}\Psi''+V\Psi$,

where $H=-\hbar^2/2m\frac{\partial}{\partial x}+V$has the form of the familiar time-independent Hamiltonian. From this, one finds that $\frac{\partial\psi}{\partial t}=\frac{-i}{\hbar}\left(-\frac{\hbar^2}{2m}\Psi''+V\Psi\right)$.

Generalizing the idea of a current from classical physics to the idea of a probability current, one takes the time derivative of the probability to get , where the product-rule for baby-math derivatives has been used and the derivative has been taken inside the integral because the integral and derivative are with respect to different variables.

Plugging in the expression for $\dot{\psi}$ from the Schrodinger's Equation, one gets

,

where the terms involving V's cancel out, and thus,

Rewriting , one can eliminate the integral in the probability current by applying the fundamental theorem of calculus (to wit: $\int_a^b \frac{\partial \psi}{\partial x} dx = \psi(b) - \psi(a)$),

$dP/dt = \frac{i\hbar}{2m}\left(-\Psi\Psi^{*\prime} + \Psi^{*}\Psi^{\prime}\right)$. But, since the probability current is usually define as $dP/dt = J(a) - J(b)$, one has

$dP/dt = \frac{i\hbar}{2m}\left(\Psi\Psi^{*\prime} - \Psi^{*}\Psi^{\prime}\right)$.

(Aside:) One can print-out a cool poster or decent T-shirt iron-on to remember the Schrodinger's Equation (among other miscellanai) at a site the current author made several years ago,
\begin{quote} http://anequationisforever.com/ds.php
\end{quote}
One can remember the general form of the probability current by recalling that it has to do with the difference of Ψ times its conjugate.

Right, so onwards with the problem:

The problem gives the wave function, so one needs just chunk out the math to arrive at the final answer,

$\Psi^{\prime} = e^{i\omega t} k\left(-\alpha \sin(kx) + \beta \cos(kx)\right)$

$\Psi^{*\prime} = e^{-i\omega t} k\left(-\alpha \sin(kx) + \beta \cos(kx)\right)$

Thus,

and,

, where one notes that the imaginary terms go to unity from the complex conjugate.

Plugging this into the probability current, one arrives at the expression for choice (E).

YOUR NOTES:

Problem 98:

Quantum Mechanics⇒}Symmetry

One recalls the simple harmonic oscillator wave functions to be symmetric about the vertical-axis (even) for the 0th energy level, symmetric about the origin (odd) for the first energy level, and so on.

If there is a wall in the middle of the well, then all the 0th energy level wave function would disappear, as would all even wave functions.

Recall the formula for SHO $E=\hbar\omega\left(n+\frac{1}{2}\right)$. The first few odd states (the ones that remain) are $E_1=3/2\hbar\omega, E_3=7/2\hbar\omega$, etc. This is choice (D).

YOUR NOTES:

Problem 99:

Quantum Mechanics⇒}Two-State Systems

Recall the mnemonic for remembering what a LASER is---Light Amplified Stimulated Emission Radiation.

A laser consists of two states, with a metastable-state in between the top and bottom state. Initially, one has all the atoms in the ground-state. But, photons come in to excite the atoms (through absorption), and the atoms jump into the top state; this is called a population inversion, as the ground-state atoms are now mostly in the top ``inverted'' state. More photons come in to excite these already excited atoms, but instead of absorption, emission occurs, and the atoms jump to a lower meta-stable state while emitting photons (in addition to the incident photons). The atoms stay in this metastable state due to selection rules, where a transition back to the ground-state is forbidden.

One doesn't need to know all that to solve this problem. Instead, merely the idea of a laser requiring two main states and a metastable state in between would suffice. Since the question gives the bottom state as $n=1$and top state as $n=3$, one deduces that the metastable state must be $n=2$, as in choice (B).

YOUR NOTES:

Problem 100:

Quantum Mechanics⇒}Raising Operator

$a^\dagger=a^*=\sqrt{\frac{m\omega_0}{2\hbar}}(x+-ip/(m\omega_0))\neq a$, and thus choice III must be true. This eliminates all but choice (C) and (E). Since one knows that the raising operator acts to raise the energy level, $[a,H]\neq 0$implies that they don't commute. This leaves just choice (C).

(Choice II is false because, from above, the condition for a Hermitian operator is violated$a^\dagger\neq a$.)

YOUR NOTES: