Solutions for class #5 from Yosumism website

Yosunism website: http://grephysics.yosunism.com

Problem 1:

Quantum Mechanics⇒}Momentum Operator

$\hat{p}=\frac{\hbar}{i}\frac{\partial}{\partial x}\psi \Rightarrow p\psi=\hbar k\psi$

YOUR NOTES:

Problem 27:

Quantum Mechanics⇒}Uncertainty

This problem looks much more complicated than it actually is. Since k and x are fourier variables, their localization would vary inversely, as in choice (B).

YOUR NOTES:

Problem 28:

Quantum Mechanics⇒}Probability

One doesn't actually need to know much (if anything) about spherical harmonics to solve this problem. One needs only the relation $P=\sum_i|\langle Y^3_i|\psi(\theta,\phi)\rangle|^2$. Since the problem asks for states where $m=3$, and it gives the form of spherical harmonics employed as Y^m_l, one can eliminate the third term after the dot-product.

So, the given wave function $\psi(\theta,\psi)=\frac{1}{\sqrt{30}}\left(5Y^3_4+Y^3_6-2Y^0_6\right)$ gets dot-product'ed, as in choice (E).

YOUR NOTES:

Problem 29:

Quantum Mechanics⇒}Bound State

Tunneling should show exponential decay for a finite-potential well, and thus choice (E) is eliminated. Choice (C) is eliminated because the wave function is not continuous. One eliminates choice (D) because the bound-state wave functions of a finite well isn't linear. The wave function for a bound state should look similar to that of an infinite potential well, except because of tunneling, the well appears larger---thus the energy levels should be lower and the wave functions should look more spread out. Choice (B) shows a more-spread-out version of a wave function from the infinite potential well.

YOUR NOTES:

Problem 50:

Quantum Mechanics⇒}Simultaneous Eigenstates

QM in verse...

Two operators, both alike in state functions,
In fair bases, where we lay our scene,
From ancient grudge break new mutiny...
Two operators unlike in eigenvalues

Yet star-crossed lovers commute.
So anyway, the problem gives $A|\alpha\rangle = \alpha|\alpha\rangle$ and $B|\alpha\rangle = \beta|\alpha\rangle$. That is, both A and B share the same eigenstate $|\alpha\rangle$.

YOUR NOTES:

Problem 51:

Quantum Mechanics⇒}Momentum

The momentum operator in position space is given by $p = \hbar / i \frac{\partial}{\partial x}$.

Thus, given the wave function, one can calculate the expectation value as <?n??? ??n>=

<?n| ?/i ?|?n>/?x ??cos(n?x/a)sin(n?x/a)dx (0,a) = 0, since sine's and cosine's are orthogonal over a whole period.

The answer is thus (A).

YOUR NOTES:

Problem 52:

Quantum Mechanics⇒}Orthonormality

$\langle \psi_m | \psi_n \rangle = \delta_{nm}$

This is the definition of orthonormality, i.e., something that is both orthogonal (self dot others = 0) and normal (self dot self = 1).

YOUR NOTES:

Problem 53:

Quantum Mechanics⇒}Energy

If one forgets the energy of an infinite well, one can quickly derive it from the time-independent Schrodinger's Equation $-\frac{\hbar}{2m}\psi''+V\psi=E\psi$. However, since $V=0$ inside, one has $-\frac{\hbar}{2m}\psi''=E\psi$.

Plug in the ground-state wave function $\psi=A\sin(kx)$, where $k=n\pi/a$. Chunk out the second derivative to get $E=\frac{k^2\hbar^2}{2m}$. Plug in k to get $E=\frac{\hbar^2 n^2 \pi^2}{2ma^2}$.

Note that $k=n\pi/a$ can be deduced from boundary conditions, i.e., the wave function vanishes at both ends ($\psi(0)=0$ and $\psi(a)=0$). The second boundary condition forces the n's to be integers. Since one can't have a trivial wave function, $|n|\geq 1$, and thus $n^2\geq 1$. One finds that $E\geq\frac{\pi^2\hbar^2}{2ma^2}$, since $n=1,2,3...$, as in choice (E).

YOUR NOTES:

Problem 56:

Quantum Mechanics⇒}Simple Harmonic Oscillator

The energy of a simple harmonic oscillator is given by $E_n=(n+\frac{1}{2})\hbar\nu$.

Thus, the ground state energy is simply $E_0=\hbar\nu/2$, as in choice (C).

YOUR NOTES:

Problem 89:

Quantum Mechanics⇒}Symmetry

There is now a node in the middle of the well. By symmetry, the ground state will disappear $n=0$, as well all the even n states. Thus, the remaining states are the odd states, as in choice (E).

YOUR NOTES:

Problem 98:

Quantum Mechanics⇒}Characteristic Equation

The characteristic equation of the matrix solves for the eigenvalues. It is $-\lambda(\lambda^2)+1=0$. Not all solutions are real, since $\lambda=e^{2i\pi/3}=\cos(2\pi/3)+i\sin(2\pi/3)$, where Euler's relation is used.

YOUR NOTES:

Problem 99:

Quantum Mechanics⇒}Perturbation Theory

The perturbed Hamiltonian is given by $\Delta H=eEz=eEr\cos\theta$, where the last substitution is made for spherical coordinates.

The first-order energy-shift is given by $\langle\psi_0|\Delta H|\psi_0\rangle$, where $\psi_0\propto e^{-kr}$.

$dV=r^2drd\Omega=r^2\sin^2\theta d\phi d\theta dr$.

Thus, $E_0=\int_0^\infty\int_0^\pi\int_0^{2\pi}e^{-kr}\cos\theta\sin^2\theta d\theta d\phi=0$, since the integral of $\cos\theta\sin^2\theta$ over 0 to π is 0.

YOUR NOTES: