Solutions for class #4 from Yosumism website

Yosunism website: http://grephysics.yosunism.com

Problem 20:

Optics⇒}Missing Fringes

Missing fringes in a double-slit interference experiment results when diffraction minima cancel interference maxima.

From a bit of phasor analysis, one can derive the diffraction factor $\beta/2=\pi w/\lambda \sin\theta$ and the interference factor $\delta/2=\pi d/\lambda \sin\theta$, where w is the width of the slits and d is the separation (taken from slit centers). The angles belong in the intensity equation given by $I \propto \sin(\beta/2)^2\cos(\delta/2)^2$.

Thus, the condition for a double-slit diffraction minimum is given by $\delta/2=m_d\pi=\pi w/\lambda \sin\theta \Rightarrow m_d\lambda=w\sin\theta$.

Also, the condition for interference maximum is given by $\beta/2=m_i\pi=\pi d/\lambda \sin\theta \Rightarrow m_i\lambda=d\sin\theta$.

Now, one needs to find the choice that allows for an integer m_d. This immediately eliminates choices (A) and (B). But, this leaves choices (C), (D), and (E). Among the remaining choices, there is only one choice that allows for slits that are smaller than the separation. This is choice (D). Take it.

YOUR NOTES:

Problem 21:

Optics⇒}Thin Film

Elimination time.

I. Can't be this, since one knows from basic thin-film theory that choice IV is right. (None of the letter choices allow for both choices I and IV.)

II. Thin film theory has $2t = \lambda/2$ for constructive interference and $2t = \lambda$ for destructive interference. Thus, the thickness of the film is smaller than that of the light. (Search on the homepage of this site for more on thin film theory---it is explained in the context of other problems.)

III. This phase change allows for the half-integer constructive interference.

IV. Phase change only occurs when light travels from a medium with lower index of refraction to a medium with higher index of refraction. Since at the back surface, the light would be going from higher to lower index of refraction, there is no phase change.

Thus, choice (E).

YOUR NOTES:

Problem 22:

Optics⇒}Telescope

The magnification for a telescope is related to the focal length for the eyepiece and objective by $M = f_o/f_e$. (Note that it is the eye-piece that magnifies it. The objective merely sends an image that's within view of the eye-piece. However, magnification is inversely related to focal length.)

The problem gives angular magnification to be $M = 10 = f_o/f_e \Rightarrow f_e = f_o/10 = .1m$. The distance between the objective and eyepiece is the sum of the focal lengths (since the light comes from infinity). $d = f_o + f_e = 1.1$m as in choice (D).

YOUR NOTES:

Problem 35:

Optics⇒}Diffraction Grating

Diffraction gratings have the same formula as 2-slit interference, except each slit is (obviously) much smaller. The condition for maximum is given by $d\sin\theta = m\lambda$, relating the width of the slit to the wavelength and angle and order m.

The width of each slit is given by the grating $d = (2000 lines/cm \times 100 cm/m)^{-1} = 0.5E-5m$. Thus, plugging in the wavelength one has $\sin\theta = \lambda/d = 5200E-10/0.5E-5 \approx 10000E-5 = 1E-1$.

Now, the approximations to get rid of the trig function. Since $\theta << 1$, one can approximate $\sin\theta \approx \theta$, where the angle is in *radians*. Now, convert the angle from radians to degrees. $1E-1 \times 180^{\circ}/\pi = 18/\pi \approx 18/3 = 6^{\circ}$, as in choice (B).

YOUR NOTES:

Problem 67:

Optics⇒}Polarized light

A plane-polarized wave has intensity $I \propto \cos^2\theta$, where θ is the angle from the wave to the polarization axis. (This is also known as Malus' Law.)

An unpolarized wave has intensity $I = const$.

Since ETS is generous enough to supply the intensity, one can easily deduce choice (C).

YOUR NOTES:

Problem 68:

Optics⇒}Aperture Formula

The formula that relates the angle of an angular aperture to the wavelength and diameter is $\theta = 1.22\lambda/d$. Thus, $d = 1.22\lambda/\theta$. Plug in numbers to get (C).

YOUR NOTES:

Problem 96:

Optics⇒}Interferometer

An (effective) path change of λ produces a fringe shift. Thus, the interferometer formula is similar to the interference formula at normal incident, $m = 2\frac{\Delta L}{\lambda}$.

Thus, $m = 2(dn/\lambda - d/\lambda) = 2d/\lambda(n-1)$. Thus, $n = \frac{m\lambda}{2d} + 1 = 1.0002$, as in choice (C).

See GR0177.100 on the same site for more info.

YOUR NOTES:

Problem 16:

Thermodynamics⇒}Carnot Engine

Recall the common-sense definition of the efficiency e of an engine,

$$e=\frac{W_{accomplished}}{Q_{input}},
$$

where one can deduce from the requirements of a Carnot process (i.e., two adiabats and two isotherms), that it simplifies to

$$e=1-\frac{T_{low}}{T_{high}}$$

for Carnot engines, i.e., engines of maximum possible efficiency. (Q_{input} is heat put into the system to get stuff going, W is work done by the system and T_{low} (T_{high}) is the isotherm of the Carnot cycle at lower (higher) temperature.)

The efficiency of the Carnot engine is thus $e=1-\frac{800}{1000}=0.2$, where one needs to convert the given temperatures to Kelvin units. (As a general rule, most engines have efficiencies lower than this.) The heat input in the system is $Q_{input}=2000J$, and thus $W_{accomplished}=400J$, as in choice (A).

YOUR NOTES:

Problem 46:

Thermodynamics⇒}Critical Isotherm

The critical isotherm is the (constant temperature) line that just touches the critical liquid-vapor region, explained in the next question. The condition for the critical isotherm is $\left(\frac{dP}{dV}\right)_c = 0$ and $\left(\frac{d^2P}{dV^2}\right)_c = 0$, where c denotes the critical point.

YOUR NOTES:

Problem 47:

Thermodynamics⇒}Liquid-Vapor Equilibrium

The liquid-vapor region is where the substance can coexist as both a liquid and vapor. (A gas is just a vapor at normal temperatures.)

In this region, the liquid and vapor are in equilibrium, hence their coexistence. Equilibrium occurs when $P_v = P_l$ and $\mu_v = \mu_l$, i.e., when the pressure and chemical potential of the liquid and vapor are equal to each other.

Since region B shows a constant pressure behavior, despite the volume-decrease, it is the region of liquid-vapor equilibrium.

YOUR NOTES:

Problem 62:

Thermodynamics⇒}Work

The work done by a gas in an isothermal expansion is related to the log of the volumes. If one forgets this, one can quickly derive it from recalling the definition of work $W=\int PdV$ and the ideal gas law equation of state $PV=nRT \Rightarrow P=nRT/V$.

One has $W=\int nRTdV/V=nRT\ln(V_1/V_0)$. For 1 mole, one has $n=1$, which yields choice (E).

(And the condition for isothermality $P_1V_1=P_0V_0=nRT_1=nRT_0$ allows one to change the argument in the log.)

YOUR NOTES:

Problem 63:

Statistical Mechanics⇒}Maximal Probability

According to statistical mechanics, maximal probability is the sate of highest entropy---it's the peak of a Gaussian curve, the average score on a normally-curved test.

Spontaneous change to lower probability thus does not occur since maximal probability is the most stable state--one of highest entropy. Boltzmann's constant never approaches 0, however in the third law of thermodynamics, one has the entropy approaching 0 for $T \rightarrow 0$.

Eliminating choices, one has choice (D).

YOUR NOTES: