Solutions for class #8 from Yosumism website

Yosumism website: http://grephysics.yosunism.com

Problem 15:

Statistical Mechanics⇒}Heat Capacity

Note that this problem wants the regime of high temperatures, and thus the answer is *not* $\frac{5}{2}R$ from classical thermodynamics, but rather $\frac{7}{2}R$.

The problem suggests that a quantized linear oscillator is used. From the energy relation $\epsilon=\left(j+\frac{1}{2}\right)\hbar\nu$, one can write a partition function and do the usual Stat Mech jig. Since one is probably too lazy to calculate entropy, one can find the specific heat (at constant volume) from $c_v=\frac{\partial U}{\partial T}\Big|_v$, where $U=NkT^2\left(\frac{\partial Z}{\partial T}\right)_V$, where N is the number of particles, k is the Boltzmann constant.

There are actually three contributions to the specific heat at constant volume. $c_v=c_{translational}+c_{rotational}+c_{vibrational}$. Chunk out the math and take the limit of high temperature to find that $c_v=\frac{7}{2}R$.

YOUR NOTES:

Problem 17:

Lab Methods⇒}Oscilloscope

This problem can be solved by elimination. Since one is given two waves, one with twice the frequency of the other, one can approximate the superposed wave (which shows up on the oscilloscope) as $\sin(\omega t)+\sin(2\omega t)$.

The summed wave no longer looks like a sine wave. Instead, it looks like a series of larger amplitude humps alternating with regions of smaller amplitudes.

However, since one is not supplied with a graphing calculator on the test, one can qualitatively eliminate the other choices based on the equation above. It is obviously not choices (D) and (E) since the superposition is still a one-to-one function. It isn't choice (C) or (B) since those are just sin waves (cosine waves are just off by a phase), and one knows that the superposed wave would look more complicated than that. Thus, one arrives at choice (A), which is a zoomed-in-view of the superposition above.

YOUR NOTES:

Problem 76:

Atomic⇒}Orbitals

The total angular momentum is given by $j=l+s$ where l is the orbital angular momentum and s is the spin angular momentum. (Note that, to an extent, l and s can be viewed as magnitudes, while m_l and m_s as directions.)

The total orbital angular momentum is just $0+1+1$, since one should recall that $(s,p,d,f)\in(0,1,2,3)$.

The spin angular momentum is just $1/2+1/2+1/2$ because one has three electrons.(Electrons are fermions that have spin $1/2$.)

Thus, the total angular momentum is $j=2+3/2=7/2$, as in choice (A).

YOUR NOTES:

Problem 77:

Quantum Mechanics⇒}Gyromagnetic Ratio

The intrinsic magnetic moment is defined in terms of the gyromagnetic ratio and spin as $\vec{\mu}_s=\gamma\vec{S}$, where $\gamma=\frac{eg}{2m}$ (g is the Lande g-factor).

Thus, one sees that the magnetic moment is inversely related to mass.

The ratio of the magnetic moment of a nucleus to that of an electron is $\mu_n/\mu_e=m_e/m_n<<1$, as in choice (E). (One can cancel out the S since ETS is nice enough to have the nucleus have the same spin as the electron.).

YOUR NOTES:

Problem 80:

Quantum Mechanics⇒}Planck Energy

The key equation is $E=hc/\lambda$. Since $hc=1.24E-6eVm$ and $E=25E3eV=2.5E4eV$, one can immediately plug the quantities in to solve for $\lambda=hc/E=1.24E-6/2.5E4=0.5E-10$, which is just choice (B).

No knowledge of X-rays required, other than the elementary knowledge that it's a electromagnetic wave and allows one to write the Planck energy as $E=hf=hc/\lambda$.

YOUR NOTES:

Problem 85:

Special Relativity⇒}Momentum

Given a total energy of $\gamma mc^2 = 1.5 MeV$ and the rest mass of the electron to be $m_e = .5 MeV/c^2$, one can figure out $\gamma = 3$.

The momentum is given by $p = \gamma mv = 3mv = 3v/2(MeV/c^2)$.

Solve for the velocity from ?= 1 / ?1 – ?^2 ??^2 = 8/9. Thus, the velocity is $v = 2c/3$.

Plugging this into the equation for momentum, one gets $p = 3/2 \times \sqrt{8/9} = \sqrt{2}$, and thus its momentum is about 1.4, as in choice (C).

YOUR NOTES:

Problem 90:

Quantum Mechanics⇒}Rotational Energy Level

Rotational energy is related to angular momentum by $E_{rot} = L^2/(2I)$. Quantum mechanics quantizes the angular momentum $L^2 = \hbar^2 l(l+1)$. Thus, $E_{rot} = \hbar^2 l(l+1)/(2I)$.

The approximate spacing between two levels is given by $\Delta E = E(l=1) - E(l=0) \approx \hbar^2/I$.

The moment of inertia of H_2 is just that of two point-masses rotating about a center-point, thus $I = 2mr^2$, taking $r = 0.5E-10m$ and $m = E-27kg$ (mass of proton). $I \approx E-47kgm^2$.

Now, $\hbar^2 \approx (6E-34)^2/(6)^2 = E-68$. Plug everything in to get the right answer. $\hbar^2/I \approx E-68/E-47 = E-21J$. Converting J to eV, one has $E-21J/E-19 = E-3$ eV, as in choice (B).

YOUR NOTES:

Problem 91:

Advanced Topics⇒}Strangeness

Elimination time:

(A) Only muons, neutrinos and electrons are leptons. Moreover, the pi-meson is a meson, which is a hadron with baryon number 0. (Hadrons interact with the strong nuclear force, while leptons interact with the weak nuclear force, em force, and possibly even the gravitation force.)

(B) The lambda has spin 1/2, as do most baryons. (The mesons have spin 0, but positive strangeness numbers.)

(C) Lepton number is already conserved, since none of the particles involved have non-zero lepton numbers. Thus, introducing a neutrino would violate (electron) lepton number conservation.

(D) No reason why...

(E) Only hadrons have non-zero strangeness (strangeness was proposed when strong particles interact as if weak particles---i.e., instead of having super-fast decay times characteristic of strong-force particles, their decay times appeared as if weak-force decays). Protons have 0 strangeness, as do pi-mesons, even though they are both hadrons. However, the lambda has -1 strangeness. Thus, strangeness is not conserved.

YOUR NOTES:

Problem 94:

Special Relativity⇒}Lorentz Transformation

Lorentz transformations are given by

$x' = \gamma(x - vt)$

$t' = \gamma(t - vx/c^2)$

Factoring out the terms, choice (C) is $x = 5/4(x - 3/5t)$, and thus $\gamma = 5/4$and $v = 3/5$. Since the equation for t fits the form above, this is a valid Lorentz Transformation.

YOUR NOTES:

Problem 95:

Advanced Topics⇒}Dimensional Analysis

The final units must be $cm^2/steradians$. One is given

$10^{12} protons/s$

$10^{20} nuclei/cm^2$

$10^{2} protons/s$

$10^{-4} steradians$

The combination $(10^2/10^{20})(1/10^{20})(1/10^{-4})$ gives the right units as well as answer choice (C).

YOUR NOTES:

Problem 97:

Advanced Topics$\Rightarrow$}Solid State Physics

This is a result one remembers by heart from a decent solid state physics course. It has to do with band gaps, which is basically the core of such a course.

Then again, one can easily derive it from scratch upon recalling some basic principles: $E=p^2/(2m)=\hbar^2k^2/(2m)$, $p=\hbar k=mv$, where k is the wave vector, E is the energy, m is the mass, and p is the momentum.

From the above, one has $dv/dt=\frac{1}{m}\frac{dp}{dt}=\frac{\hbar}{m}\frac{dk}{dt}$.

Set the two dv/dt's equal to get $\frac{\hbar}{m}\frac{dk}{dt}=\frac{1}{\hbar}\frac{d^2E}{dtdk}$. Cancel out the dt's to get $\frac{\hbar^2}{m}dk=\frac{dE}{dk}\Rightarrow m=\hbar^2/(\frac{dE}{dk})$, after differentiating with respect to k on both sides.

Alternatively, one can try it Kittel's way:

Start with $\hbar v_g=dE/dk$. Then, $dv_g/dt=\hbar^{-1}(d^2E/dt^2dk/dt)=\hbar^{-1}(d^2E/dt^2F/\hbar)$. Thus, the effective mass is defined by $F=\hbar^2/(d^2E/dk^2)dv_g/dt=mdv_g/dt$.

YOUR NOTES: