Solutions for class #15 from Yosumism website

Yosumism website: http://grephysics.yosunism.com

Problem 66:

Mechanics⇒}Effective Potential

One can solve this problem by remembering the effective potential curve $V_{eff}(r)=V(r)+L^2/(2mr^2)$. For the gravitational potential, one has $V(r)\propto -1/r$.

The total energy of the spaceship is $E_s=1/2m(1.5v_J)^2+V_{eff}$, while the total energy of Jupiter is $E_J=1/2m(v_J)^2+V_{eff}$

(A) A spiral orbit occurs when $E<V_{min}$, which corresponds to $v_s<<v_J$.

(B) A circular orbit occurs only when $E=V_{min}$. Since the energy of Jupiter is greater than that of the spaceship--and (see below) since Jupiter itself has $E>V_{min}$, the spaceship must have $E>V_{min}$.

(C) An ellipse occurs for $V_{min}<E<0$. Planets orbit in ellipses. However, since the speed of the ship is greater than Jupiter's orbit speed by a good bit, one assumes its total energy is $E>0$.

(D) A parabolic orbit occurs for $E=0$. The condition is much too stringent.

(E) A hyperbolic orbit occurs for $E>0$. See (C). Since $E_s>0>E_J$, this is it.

YOUR NOTES:

Problem 68:

Mechanics⇒}Lagrangians

The potential energy of the mass is obviously $U = mgs\cos\theta$, and thus one eliminates choices (B) and (C). (To wit: $L = T - U$, (C) has the wrong sign).

The translational part of the kinetic energy is easily just $1/2m\dot{s}^2$. The rotational part requires the calculation of the moment of inertia for a point particle, which is just $I = mr^2$, where $r = s\sin\theta$, in this case. Thus, the rotational kinetic energy is $1/2m(s\sin\theta)^2\omega^2$. The only choice that has the right rotational kinetic energy term is choice (E).

YOUR NOTES:

Problem 80:

Mechanics⇒}Wave Phenomena

There's a long way to solve this problem and then a short. One looks at the choices to find the one that first the physical deduction: when $\mu_l = \mu_r$, the whole incident wave should be transmitted, with 0 reflection. Moreover, in the limit of $\mu_r >> \mu_l$ there should be 0 transmission. Choice (C) is the only one that fits this condition, leading to a ratio of 1 for $\mu_l = \mu_r$.

One can also calculate the exact form of the transmission coefficient for this multi-density string.

At the boundary between different density parts, one applies continuity $\psi_i(x=0)+\psi_r(x=0)=\psi_t(x=0)$ to get 1+R=T.

One applies $m\ddot{\psi}=0=\frac{\partial\psi}{\partial x}(x<0)-\frac{\partial\psi}{\partial x}(x>0)$, where $m=0$ since there is no point particle situated at the origin, to obtain $ik_l(1-R)=ik_rT$.

Recalling the nifty relation $\omega=ck$ and $c=\sqrt{F/\mu}$, one solves for T to get $T=\frac{2k_l}{k_r+k+l}=\frac{\sqrt{\mu_l/\mu_r}}{1+\sqrt{\mu_l/\mu_r}}$, as in choice (C).

YOUR NOTES:

Problem 81:

Wave Phenomena⇒}Beats

One remembers the relation for beats, i.e., $f_1 - f_2 = f_{beat}$. Beat phenomenon occurs when two waves occur at nearly the same frequency.

Taking $f_0 > \approx 73$ as the fundamental frequency, one deduces the harmonic to be $440/f_0 \approx 6$. $440 - 6f_0 \approx 0.5$, and thus the answer is choice (B).

One can derive the relation for beats by recalling the fact that one gets beat phenomenon when one superposes two sound waves of similar frequency $f_1 \approx f_2$, say, of the form $A\sin(2\pi f_i t)$,

$$f = A\sin(2\pi f_1 t) + A\sin(2\pi f_2 t) = 2A\sin(2\pi(f_1 + f_2)/2t)\cos(2\pi(f_1 - f_2)/2t),$$

where to get beats phenomenon one must have $\cos(t(f_1 - f_2)/2) = \pm 1 \Rightarrow 2\pi = (f_1 - f_2)/2t$, and since there are two beats per period, one has $f_1 - f_2 = f_{beat}$.

YOUR NOTES:

Problem 83:

Mechanics⇒}Rippled Surface

The simple intuitive way to solve this is to note that for $d \rightarrow \infty$, $v \rightarrow 0$, since one gets an infinitely steep (vertical line) hill, and the only way for the particle to stay on the surface (i.e., not accelerate on it) at the vertical drop is if its velocity is 0. The onyl choice with d on the denominator is choice (D).

The more rigorous solution is due to Sara Salha.

Equating centripetal force with gravity at the top of the hill, one has $mv^2/r = mg \Rightarrow v = \sqrt{mgr}$. The non-trivial bit comes from calculating the radius.

Recall the radius of curvature from calculus $1/r = \kappa = \frac{|\ddot{x}\dot{y} - \ddot{y}\dot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}}$. Defining a parameter t as the independent variable, and defining $x = t$, $y = d\cos(kt)$, one finds that $1/4 = \frac{k^2 d\cos(kt)}{1+(kd)^2\sin^2(kt)}$. Evaluate it at $t = 0$ to find $1/r(0) = k^2 d \Rightarrow r = 1/(k^2 d)$, the radius of curvature at the top of the hill. Plug that into the equation for velocity above to get $v = \sqrt{\frac{mg}{kd^2}}$, as in choice (D).

YOUR NOTES:

Problem 84:

Mechanics⇒}Normal Mode

One can work through the formalism of the usual normal mode analysis or learn how to deal with normal mode frequencies the easy way:

The highest normal mode frequency is due to the two masses oscillating out of phase. The ω^2 contribution from the pendulum is just g/l. The ω^2 contribution from each mass due to the spring is k_i/m_i. This is choice (D).

(If one had to guess, one can immediately eliminate choice (A), since that is the lowest normal mode frequency. In normal modes, there's always an in-phase frequency, which tends to be the lowest frequency.)

YOUR NOTES:

Problem 85:

Mechanics⇒}Wave Phenomena

One can solve this problem without knowing anything about mechanics (but with just the barest idea of wave phenomenon theory). Following the hint, one considers the limiting cases:

$M \to \infty \Rightarrow \mu/M \to 0$... With an infinite M, the string is basically fixed on the rod, and its wavelength is just $\lambda = L$. One eliminates choice (A) from the fact that $\cos 2\pi = 1 \neq 0$, as μ/M demands in this regime.

$M \to 0 \Rightarrow \mu/M \to \infty$... Without the presence of the mass M, one has $\lambda = 4L$. Thus, $2\pi L/\lambda = \pi/2$. Since $\tan x = \sin x/\cos x$ and $\cos \pi/2 = 0$, one finds that choice (B) is the only one that fits this condition.

YOUR NOTES:

Problem 92:

Mechanics⇒}Potential

Given $V(x) = -ax^2 + bx^4$, one can find the minimum by taking the first derivative (second derivative test indicates concave up), $V'(x) = \left(-2ax + 4bx^3\right)_{x_0} = 0 \Rightarrow x_0 = \sqrt{\frac{a}{2b}}$.

The force is given by $F = -dV/dx = 2ax - 4bx^3$.

The angular frequency of small oscilations about the minimum can be found from,

F(X-Xo) = 2a(X-Xo) – 4b(X-Xo)^3

= (2a – 12bXo^2)X

mX” = -4aX

X” = -w^2X   w^2 =4a/m

where one might recall the binomial theorem or pascal's triangle to quickly figure out the trinomial coefficients.

One finds that $\omega = 2\sqrt{\frac{a}{m}}$, as in choice (D).

YOUR NOTES:

Problem 93:

Mechanics$\Rightarrow$}Potential

The problem gives a nifty potential energy graph. The period is due to each part of the potential graph.

For the simple harmonic oscillator (SHO) part, one remembers the formula $T = 2\pi\sqrt{m/k}$ (to wit: $m\ddot{x} = -kx \Rightarrow \ddot{x} = -\omega^2 x \Rightarrow \omega = \sqrt{k/m} = 2\pi f = 2\pi/T \Rightarrow T = 2\pi\sqrt{m/k}$). However, that period is for a particle to oscillate from one end of the potential curve to the other end and then back again. Since the graph shows only half of the usual SHO potential, the period contribution from the SHO part should be half the usual period: $T_{SHO_{1/2}} = \pi\sqrt{m/k}$

For the gravitational potential, one can calculate the period from the usual kinematics equation for constant acceleration. Recall the baby-physics equation, $x = \sqrt{1}2gt^2 \Rightarrow t = \sqrt{2x/g}$. The quantity needs to be converted to the relevant parameters of the problem. The problem supplies the constraint that the energy is constant, $E=\frac{1}{2}mv^2+mgx$. At the endpoint, one has $v=0\Rightarrow x=E/mg$. Plugging this into the equation for time, one gets $t=\sqrt{2E/mg^2}$. Since the particle has to travel from the origin to the right endpoint and then back to the origin, the total time contribution from this potential is twice that, $T_{grav}=2\sqrt{2E/mg^2}$.

The total period is thus the sum of the above contributions, which is choice (D).

YOUR NOTES: