Solutions for class #11 from Yosumism website

Yosumism website: http://grephysics.yosunism.com

Problem 5:

Mechanics$\Rightarrow$}Centripetal Force

$\vec{F}_{air}$acts in the direction as shown and the centripetal acceleration acts in the direction of $\vec{F}_A$. Centripetal acceleration is a net force, however, and thus,

$$\rightarrow_+ \sum F_x = 0 = -F_{air} + f_x$$
$$\uparrow_+ \sum F y = -mv^2/r = f_y
$$

$f_x$is in the positive direction and $f_y$is in the negative direction. Thus the force of the road is $\vec{F}_B$.

YOUR NOTES:

Problem 6:

Mechanics$\Rightarrow$}Inclined Plane

Set up the usual coordinate system with horizontal axis parallel to incline surface. The equations are, (since the mass slides down at constant speed),

$$\sum F_x = 0 = f - mg\sin\theta$$
$$\sum F y = 0 = N - mg\cos\theta
$$

Friction is given by $f = \mu N = \mu mg\cos\theta$, where the normal force $N$is determined from the F_y equation. For constant velocity one also has, $f = mg\sin\theta = \mu mg\cos\theta \Rightarrow \mu = \tan\theta$

To find the work done by friction, one calculates $W = fL$, where $L\sin\theta = h$. Thus $W = \tan\theta mg\cos\theta\frac{h}{\sin\theta} = mgh$, as in the almost-too-trivial, but right, choice (B).

YOUR NOTES:

Problem 7:

Mechanics⇒}Elastic Collisions
One determines the velocity of impact of the ball from conservation of energy,

$$mgh=\frac{1}{2}mv_0^2\Rightarrow v_0^2=2gh$$

Conservation of momentum gives,

$$v_0=-v_1+2v_2$$

Conservation of *kinetic* energy gives,

$$mv_0^2=mv_1^2+2v_2^2$$

Plug in the momentum and kinetic energy conservation equations to solve for $v_1$and $v_2$in terms of $v_0$to get

$$v_1 \quad = \quad -v_0/3$$
$$v_2=2v_0/3$$

Write yet another conservation of energy equation for the *final* energy,

$$\frac{1}{2}mv_1^2+\frac{1}{2}2mv_2^2=mgh'+\frac{1}{2}2mv_2^2,$$

where the condition that the mass $2m$ slides on a *frictionless* plane is used.

Thus, $\frac{1}{2}mv_1^2=\frac{v0^2}{18}=mgh'\Rightarrow h'=\frac{h}{9}$, where the previous result $v_1=-v_0/3$and $v_0^2=2gh$is used.

YOUR NOTES:

Problem 20:

Mechanics⇒}Conservation of Momentum

The Helium atom (m) makes an elastic collision, and thus the kinetic energy before and after is conserved.

$\frac{1}{2}mv^2 = \frac{1}{2}m(0.6v)^2 + \frac{1}{2}MV'^2 \Rightarrow 0.64mv^2 = MV'^2$

Conservation of momentum requires that,

$mv = -.6mv + MV' \Rightarrow V' = 1.6mv/M$

From kinetic energy conservation,
$0.64mv^2 = MV'^2 \Rightarrow 0.64mv^2 = (1.6mv)^2/M \Rightarrow 0.64 = 1.6^2 m/M \Rightarrow M = 1.6^2 m/0.64 = 4m$, but since $m = 4u$, $M = 16u$, for O_2, as in choice (D).

YOUR NOTES:

Problem 21:

Mechanics⇒}Moment of Inertia

To solve this problem, one should remember the parallel axis equation to calculate the moment of inertia about one end of the hoop:
$I = I_{cm} + md^2 = mR^2 + md^2 = 2mR^2$,
where d is the distance from the pivot point to the center of mass, which in this problem, is just equal to R. (In the last equality, note that the moment of inertia of a hoop of radius R and mass m about its center of mass is just $I_{cm} = mR^2$.)

The problem gives the period of a physical pendulum as $T = 2\pi\sqrt{I/(mgd)}$. Thus, plugging in the above result for the moment of inertia, one has,
$T = 2\pi\sqrt{2mR^2/(mgR)} = 2\pi\sqrt{2R/(g)} \approx 2*3\sqrt{2*0.2/(10)} = 12/10 = 1.2s$, which is closest to choice (C). (Since π was rounded to 3, the period should be slightly longer than 1.2s.)

YOUR NOTES:

Problem 22:

Mechanics⇒}Geometry

The harder part of this problem involves determining the radius of Mars. It's an approximate geometry problem. The problem gives a vertical drop of 2m for every 2600m tangent to the surface. The tangent to the surface is approximately one leg of a triangle whose hypotenuse is the radius of Mars, since the radius is much larger than the tangent distance. The other leg of the right triangle is just $r-2$, where r is the radius of Mars. In equation form, what was just said becomes $(r-2)^2+3600^2=r^2$. The square terms cancel out, and dropping out the 4, one has $r \approx 3600^2/2 \approx 8E6m$.

(The above deduction was due to Ayanangsha Sen.)

The easier part comes in the final half of the problem: applying the centripetal force to the force of gravity. $mv^2/r=mg \Rightarrow v=\sqrt{2gr} \approx \sqrt{20*8E6}=\sqrt{16E7} \approx 4000m/s$, which is closest to $3.6km/s$, as in choice (C).

YOUR NOTES:

Problem 23:

Mechanics⇒}Stability of Orbits

The gravitational force suspect to a bit of perturbation is given as $\vec{F}_{12}=\hat{r}_{12}Gm_1m_2/r_{12}^{2+\epsilon<i>n</i>}$.

One can narrow down most choices by recalling some basic facts from central force theory:

(A) No mention is made of frictional effects, and thus energy should be conserved.

(B) Angular momentum is always conserved since the net torque is 0 (to wit: the force and moment arm are parallel).

(C) This is just Kepler's Third Law applied to this force. (Recall the following bromide: The square of the period is equal to the cube of the radius---for the inverse square law force. For a perturbed force, the bromide becomes: The square of the period is equal to the $3+\epsilon$ power of the radius.)

(D) Recall Bertrand's Theorem from Goldstein. Stable non-circular orbits can *only* occur for the simple harmonic potential and the inverse-square law force. This is of neither form, and thus this choice is FALSE.

(E) Circular orbits exist for basically all potentials. A stationary orbit exists if and only if the following conditions are satisfied: $V'=0 \quad V''>0$. Recall that the potential is related to the force by $-V'=F \Rightarrow V=-\int F dx$. Use $V \propto 1/r^n$, and recalling the extra term added to the effective potential to be $L^2/(2mr^2)$, one chunks out the derivatives to get the condition that $n<2$, as a potential exponent, ($n<3$, as a force exponent) for stable orbit. One can remember this result or re-derive it whenever necessary. For $n<3$, (the power exponent of the force equation), a stable circular orbit exists. Since ϵ is presumably less than 1, the planet does, indeed, move in a stationary circular orbit about the sun.

YOUR NOTES:

Problem 30:

Advanced Topics$\Rightarrow$}Fluid Mechanics

Equipotential leads to equipressure in a fluid. Thus, take the pressure at the base of the dark fluid and set it equal to the pressure (of the lighter-colored fluid) at a horizontal-line across on the right-hand side of the U:

$P_{dark}=\rho_4 g(5)=P_{light}=\rho_1 g(h_2-(h_1-5))=\rho_1 g(h_2-h_1+5)$. The initial total height of the columns is 40, thus after the darker liquid is added, the total height is 45. Plug $h_1+h_2=45$ into the equation above to get $h_1=15$, $h_2=30$, and therefore $h_2/h_1=2/1$, as in choice (C).

(Ah, one should remember that the fluid pressure at a point is due to all the water on top of it, thus $P=\rho g h$, where h is the height of the water on top of the point.)

YOUR NOTES:

Problem 31:

Mechanics⇒}Frictional Force

(A) A falling object experiencing friction falls faster and faster until it reaches a terminal speed. Its kinetic energy increases proportional to the square of the velocity and approaches a asymptotic value.

(B) The kinetic energy increases to a maximum, but it does not decrease to 0. See (A).

(C) The maximal speed is the terminal speed.

(D) One has the equation $m\ddot{y}+b\dot{y}+mg=0 \Rightarrow m\dot{v}+bv+mg=0$. Without having to solve for v, one can tell by inspection that $v(t)$will depend on both $b$and m.

(E) See (D). This is the remaining choice, and it's right.

YOUR NOTES:

Problem 32:

Mechanics⇒}Moment of Inertia

The inertia through the point A is $I_A=3mr^2$. From geometry, one deduces that the distance between each mass and the centerpoint A is $r\cos(30^\circ)=l/2 \Rightarrow r=l/\sqrt{3}$. T he moment of inertia about A is thus $I_A=ml^2$

The inertia about point B can be obtained from the parallel axis theorem ($I_{displaced}=I_{cm}+\sum_i m_i d^2$, where d is the displaced distance from the center of mass). Because $d=l/\sqrt{3}$, one has $I_B=I_A+3md^2=2I_A$. Since the angular velocity is the same for both kinetic energies, recalling the relation for kinetic energy $K_i=I_i\omega_i^2$, one has $K_B/K_A=I_B/I_A=2$, as in choice (B).

YOUR NOTES:

Problem 44:

Mechanics⇒}Chain Rule

Recall that $a=\frac{dv}{dt}=\frac{dv}{dx}\frac{dx}{dt}=v\frac{dv}{dx}.\ \frac{dv}{dx}=-n\beta x^{-n-1}\Rightarrow v\frac{dv}{dx}=-n\beta^2x^{-2n-1}$, as in choice (A).

YOUR NOTES:

Problem 65:

Mechanics⇒}Conservation Laws

From conservation of momentum, one has $mv_0=Mv\Rightarrow v_0=Mv/m$. The man does work on both himself and the boat. Thus, the work-kinetic energy theorem has $W=\Delta K=1/2mv_0^2+1/2Mv^2=1/2(M^2/m+M)v^2, as in choice (D)$

YOUR NOTES: