Solutions for class #7 from Yosumism website

Problem 44:

Mechanics⇒}Lagrangians

The kinetic energy, in general, is given by $T=\frac{1}{2}m(\dot{x}^2+\dot{y}^2)$. The potential energy is just $V=mgy$. The Lagrangian is given by $L=T-V=\frac{1}{2}m(\dot{x}^2+\dot{y}^2)-mgy$.

Now, given the constraint $y=ax^2$, one can differentiate it and plug it into the Lagrangian above to reexpress the Lagrangian in terms of just y, for example.

Differentiating, one has $\frac{d}{dt}(y=ax^2)\Rightarrow\dot{y}=a2x\dot{x}\Rightarrow\dot{x}=\dot{y}/(2ax)$. Square that to get $\dot{x}^2=\frac{\dot{y}^2}{4a^2x^2}=\frac{\dot{y}^2}{4ay}$, where one replaces the x^2through the given relation $y=ax^2$.

Plug that back into the Lagrangian above to get exactly choice (A).

YOUR NOTES:

Problem 45:

Mechanics⇒}Conservation of Energy

Conservation of energy gives $mgh=\frac{1}{2}mv_0^2$, where $v_0$is the velocity of the ball before it strikes the ground. Thus, $v_0^2=2gh$.

Afterward, the ball bounces back up with $v'=0.8v$. Apply conservation of energy again to get $mgh'=\frac{1}{2}mv'^2 \Rightarrow v'^2=2gh' \Rightarrow h'=v'^2/(2g)$.

Plugging in $v'^2=0.8^2v_0^2$, one has $h'=0.64h$, which is choice (D).

YOUR NOTES:

Problem 61:

Mechanics⇒}Small Oscillations

One can derive the frequency of small oscillation for a rigid body in general by using the torque form of Newton's Laws: $\tau = I\ddot{\theta} = \vec{r} \times \vec{F}$. (I is moment of inertia, r is moment arm)

In this case, one has a constant downwards force $F = mg$, which acts at a moment arm angle θ. Thus, $I\ddot{\theta} = -rmg\sin\theta \approx -rmg\theta$, where the approximation works if $\theta << 1$.

The equation of motion for small angles is thus $\ddot{\theta} = -(mgr/I)\theta$. This is similar in form to that of a simple harmonic oscillator with the angular frequency being $\omega = \sqrt{mgr/I}$.

Now, the problem is to find the angular frequency for each system.

One needs not worry about the rod, since it is massless and has no moment of inertia.

The moment of inertia of system I is just $I_I = 2mr^2$. The radius of gyration r is just $r_I = 2r$ (an r for each mass).

The moment of inertia of system II is $I_{II} = mr^2 + m(r/2)^2 = \frac{5}{4}mr^2$. The radius of gyration r is just $r_{II} = r/2 + r = 3r/2$.

Thus $\omega_{II}/\omega_I = \sqrt{(r_{II} I_I)/(r_I I_{II})} = \sqrt{2 \times 3r/2/(2(5/4) \times r)} = \sqrt{6/5}$, as in choice (A).

YOUR NOTES:

Problem 66:

Mechanics⇒}Work

Work is defined by $W=\vec{F}\cdot d\vec{l}$.

The force here is just due to gravity, thus $F=\rho y g$, where $\rho=2kg/m$ is the density of the chain. The chain is wound upwards, so work is $W=\int_0^{10}\rho g y dy=\frac{2}{2}(g x^2)_0^{10}=10\times 100=1000J$, as in choice (C). (The approximation $g\approx 10m/s^2$ is made.)

YOUR NOTES:

Problem 74:

Mechanics⇒}Small Oscillations

The small oscillations of the hoop has the same frequency as that of a simple pendulum. Thus, $\omega^2=\frac{g}{l}$. However, in this case, l is the distance from the center of mass to the oscillation point---which is just the radius of the loop.

Since $\omega=2\pi/T$, the period $T\propto\sqrt{\frac{r}{g}}\propto\sqrt{r}$ does not depend on mass.

Since $r_x=4r_y$. The ratio of periods is $T_X/T_Y T/\sqrt{r_y}T/\sqrt{r_x/4}2$. Thus, the period of Y is just $T/2$. (Note, the technique of leaving out constants requires that mimeTeX failed to render your expression 's are used instead of $=$'s. Practice a few times with this technique, as this will save time on the actual exam.)

YOUR NOTES:

Problem 78:

Mechanics$\Rightarrow$}Multiple Particles

The angular momentum equation gives the angular frequency. $L=I\omega=m\vec{r}\times\vec{v}$, which relates the angular momentum to the moment of inertia, the angular velocity, the radius of gyration and the linear velocity.

The system spins about its center of mass, which is conserved. Since the pole is massless and the skaters are off the same mass, $r_{cm}=b/2$. The moment of inertia of the system is just $I=2mr_{cm}^2=2m(b/2)^2$.

Thus, the angular momentum equation gives , since the cross-products point in the same direction. Now that one has the angular velocity, one eliminates all but choices (B) and (C).

Now, take the time-derivative of x for choices (B) and (C), then evaluate it at $t=0$.

For B, one has $dx/dt=v+1.5v\cos(3vt/b)\rightarrow 2.5v$ for $t=0$.

For C, one has $dx/dt=0.5v+1.5v\cos(3vt/b)\rightarrow 2v$ for $t=0$.

Since the top skater is initially at $v(0)=2v$, only choice (C) has the right initial condition. Choose choice (C).

(FYI: The center of mass velocity is given by $v_{cm}=\frac{2mv-mv}{2m}=v/2$. One can also arrive at (C) by noting conservation of center of mass velocity, since there is no net force.)

YOUR NOTES:

Problem 82:

Mechanics⇒}Torque

The problem gives $H = \int \tau dt = I\alpha t$, but $\omega = \alpha t$. Thus, $\omega = H/I$.

The moment of inertia of a plate about the z-axis is just $1/3Md^2$. Plug this into ω to get choice (D).

YOUR NOTES:

Problem 87:

Mechanics⇒}Conservation of Energy

Do not immediately try applying the Virial Theorem to this one. Instead, consider conservation of energy. Coming in from far away, the particle has $E = V = 0$the total energy equal to the potential energy equal to 0.

Alternatively, one has, for a circular orbit, the equality between centripetal force and the attractive force, $mv^2/r = K/r^3 \Rightarrow v^2 = K/(mr^2)$.

Thus, the kinetic energy is just $T = p^2/(2m) = K/(2r^2)$.

Since $V = -\int F dr = -K/(2r^2)$, where the extra negative sign for the potential energy is due to an attractive potential.

Thus, the total energy $E = T + V = 0$, which is choice (C).

(Alternate solution is due to user crichigno.)

YOUR NOTES:

Problem 93:

Mechanics⇒}Boundary Condition

Getting low on time, one should begin scoring points based more of testing strategy than sound rigorous physics. At the initial release point, the acceleration is due to gravity and the tension is 0 (no centripetal acceleration). The only choice that gives $a(\theta = 0) = g$ is choice (E).

YOUR NOTES:

Problem 100:

Mechanics⇒}Sum of Moments

Take the sum of the moments (or torque) about the triangular pivot fulcrum and set it to 0.

$\sum M = 20gd + 20gq - 40gx = 0$, where d is the distance from the fulcrum to the 20kg mass, x is the distance from the fulcrum to the 40kg mass and q is the distance from the fulcrum to the center of mass of the rod.

From conservation of length, one also has $d + x = 10$ and $q + x = 5$.

Plug everything into the moment equation. Shake and bake at 300 K. Solve for q to get choice (C).

Alternatively, one can solve this problem in one fell swoop. Taking q as the distance from the fulcrum to the center of mass of the rod, one sums the moment about the fulcrum to get $\sum M = 20gq + 20g(5+q) - 40(5-q)g = 0$. Solve for q to get choice (C). (This is due to the user astro_allison.)

YOUR NOTES: