Solutions for class #1 from Yosunism website

Yosunism website: http://grephysics.yosunism.com

Problem 4.

Mechanics $\Rightarrow$}Gravitational Law

Recall the famous inverse square law determined almost half a millennium ago,

$F = \frac{k}{r^2}$,

where $k = GMm$.

The ratio of two inverse-square forces ($r > R$, where R is the radius of the planet or huge heavy object) would be

$\frac{F(r_1)}{F(r_2)} = \frac{4r_2^2}{r_1^2}$.

Thus, $\frac{F(R)}{F(2R)} = \frac{4R^2}{R^2} = 4$, which is choice (C).

YOUR NOTES:

Problem 5.

Mechanics$\Rightarrow$}Gauss Law

The inverse-square law doesn't hold inside the Earth, just like how Coulomb's law doesn't hold inside a solid sphere of uniform charge density. In electrostatics, one can use Gauss Law to determine the electric field inside a uniformly charged sphere. The gravitational version of Gauss Law works similarly in this mechanics question since $\nabla\cdot\vec{E}=\rho_e\Rightarrow\nabla\cdot\vec{g}=\rho_M$, where ρ_M is the mass density of M. In short, the gravitational field $\vec{g}$ plays the analogous role here as that of $\vec{E}$ Thus, $\oint\vec{g}\cdot d\vec{a}=\int\rho dV$.

So, for $r<R$, $g(4\pi r^2)=\rho\frac{4}{3}\pi r^3\Rightarrow g=r\frac{\rho}{3}$, where one assumes ρ is constant.

To express the usual inverse-square law in terms of ρ, one can apply the gravitational Gauss Law again for $r>R$, $g(4\pi r^2)=\rho\frac{4}{3}\pi R^3\Rightarrow g=\frac{R^3}{r^2}\frac{\rho}{3}$

Since $\vec{F}=m\vec{g}$ Therefore,

$$\frac{F(R)}{F(R/2)}=2.$$

which is choice (C).

YOUR NOTES:

Problem 6.

Mechanics ⇒}Method of Sections

By symmetry, one can analyze this problem by considering only *one* triangular wedge. The normal force on one wedge is just $N = (m+M/2)g$, since by symmetry, the wedge (m) carries half the weight of the cube (M). The frictional force is given by $f = \mu N = \mu(m+M/2)g$.

Sum of the forces in the horizontal-direction yields $F_x = 0 \le f - N_M/\sqrt{2} = \mu(m+M/2)g - Mg/2$ for static equilibrium to remain valid. (Note that the normal force of the cube is given by $N_M = Mg/\sqrt{2}$since, summing up the forces perpendicular to the plane for M, one has, $N_M \sin(\pi/4) = Mg/2$. Also, note that it acts at a 45 degree angle to the wedge.)

Solving, one has $\mu(m+M/2)g \ge Mg/2 \Rightarrow M \le \frac{2\mu m}{1-\mu}$.

(In a typical mechanical engineering course, this elegant method by symmetry is called the *method of sections*.)

YOUR NOTES:

Problem 7.

Mechanics $\Rightarrow$}Normal Modes

For normal mode oscillations, there is *always* a symmetric mode where the masses move together as if just one mass.

There are three degrees of freedom in this system, and ETS is nice enough to supply the test-taker with two of them. Since the symmetric mode frequency is not listed, choose choice it!---as in (A).

YOUR NOTES:

Problem 8.

Mechanics $\Rightarrow$}Torque

The problem wants a negative z component for τ. Recall that $r \times F = 0$ whenever r and F are parallel (or antiparallel). Thus, choices (A), (B), (E) are immediately eliminated. One can work out the cross-product to find that (D) yields a positive τ_z, thus (C) must be it.

YOUR NOTES:

Problem 19.

Mechanics ⇒}Mass of Earth

If one does not remember the mass of the earth to be on the order of $10^{24} kg$, one might remember the mass of the sun to be $10^{30} kg$. Since the earth weighs much less than that, the answer would have to be either (A) or (B). The problem gives the radius of the earth, and one can assume that the density of the earth is a few thousand $kg/m^3$and deduce an approximate mass from $m=\rho V$. The answer comes out to about 10^{22}, which implies that the earth is probably a bit more dense than one's original assumption. In either case, the earth *can't* be, on average, uniformly $10^9 kg/m^3$ dense. Thus (A) is the best (and correct) answer.

An alternate solution is provided by the user SlickAce21. Equating the mass of some object with the gravitational force, one has
$mg=GmM/r^2 \Rightarrow g=GM/r^2 \Rightarrow M=gr^2/G \approx 6E24$.

YOUR NOTES:

Problem 39.

Advanced Topics⇒}Fourier Series

There's no need to go through the formalism of integrating out the coefficients.

One can tell by inspection that the function is odd. Thus, one would use the Fourier sine series. This leaves choices (B) and (A).

Choice (A) is trivially zero since for all integer n, $\sin(n\omega t)=0$. Choice (B) remains.

YOUR NOTES:

Problem 40.

Mechanics ⇒}Centripetal Force

There is no tangential acceleration (since otherwise it would slide and not roll---the frictional force balances the forward acceleration force). However, there is a centripetal acceleration that pulls the particles back in a circle, as in choice (C). This acceleration propels the tangential velocity to continue spinning in a circle.

YOUR NOTES:

Problem 41.

Mechanics ⇒}Energy

The kinetic energy is related to the inertia I and angular velocity ω by $K = \frac{1}{2} I \omega^2$.

The problem supplies $I = 4 kgm^2$ so one needs not calculate the moment of inertia. The angular velocity starts at $80 rad/s$ and ends at $40 rad/s$.

Thus, the kinetic energy lost $\Delta K = \frac{1}{2} I(\omega_f^2 - \omega_0^2) = \frac{1}{2}(4)(40^2 - 80^2) = 2(1600 - 6400) = -9600 J$, as in choice (D).

YOUR NOTES:

Problem 42.

Mechanics $\Rightarrow$}Angular Kinematics

Kinematics with angular quantities is exactly like linear kinematics with

$x \to \theta$ (length to angle)

$a \to \alpha$(linear acceleration to angular acceleration)

$v \to \omega$(linear velocity to angular velocity)

$m \to I$(mass to moment of inertia)

$F \to \tau$(force to torque).

Thus, one transforms $v = v_0 + at \Rightarrow \omega = \omega_0 + \alpha t$.

Plugging in the given quantities, one gets $\alpha = \frac{\omega - \omega_0}{t} = \frac{40-80}{10} = -4 rad/s^2$.

The torque is given by $\tau = I\alpha = -16 Nm$, whose magnitude is given by choice (D).

YOUR NOTES:

Problem 43.

Mechanics ⇒}Lagrangians

Recall the Lagrangian equations of motion $\frac{\partial L}{\partial q} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}$.

If $\frac{\partial L}{\partial q} = 0$ then $\frac{\partial L}{\partial \dot{q}} = constant$, since its time-derivative is 0.

One can relate energy to momentum from elementary considerations by $\frac{\partial L}{\partial \dot{x}} = m\dot{x}$, where L is the kinetic energy $0.5m\dot{x}^2$. Thus, the generalized momentum defined for a generalized coordinate is just $p_n = \frac{\partial L}{\partial \dot{x}}$.

From the above deductions, the generalized momentum is constant, as in choice (B).

(Incidentally, the ignorable or cyclic coordinate would be q_n and not p_n since it does not appear in the Lagrangian.)

YOUR NOTES: