

Solutions from Yosumism website

Yosumism website: <http://grephysics.yosunism.com>

Problem 61

Electromagnetism ⇒ } Gauss Law

Recall Gauss Law $\oint \vec{E} \cdot d\vec{A} = q/\epsilon_0$. Thus, $E(4\pi r^2) = \int_0^{R/2} 4\pi r^2 \rho dr = 4\pi A(R/2)^3/\epsilon_0$. Solving for E, one has $E = A/\epsilon_0(R/2)^3$, as in choice (B).

YOUR NOTES:

Problem 62:

Electromagnetism ⇒ } Capacitors

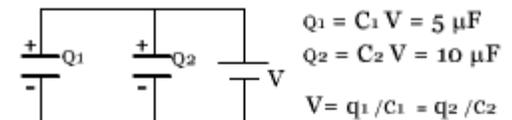
Initially, one has the two capacitors connected in parallel, so that each receives the same voltage from the battery. Thus $V=5=Q_1/C_1=Q_2/C_2$. $C_1=1$ and $C_2=2$, thus $Q_1=C_1V=5$ and $Q_2=C_2V=10$.

After the battery is removed and the capacitors are re-connected so that the opposite plates face each other, one has (immediately) $Q_1 - Q_2 = -5$. The charges would then redistribute themselves so that the voltage across each capacitor is the same. Thus, denoting the final charge on each capacitor as q_1, q_2 , respectively, one has (from charge conservation) $-5 = q_1 + q_2$. Applying the equi-voltage condition, one has $q_1/C_1 = q_2/C_2 \Rightarrow q_1 = q_2 C_1/C_2$. Plug that into the charge conservation equation to get $-5 = (1 + C_1/C_2)q_2 \Rightarrow q_2 = -3.33 \Rightarrow V = q_2/C_2 \approx 1.7$, as in choice (C). (As an exercise, one can also check by computing the charge for the other capacitor.)

YOUR NOTES:

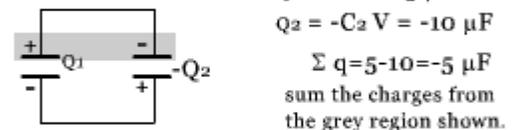
initial state

The capacitors are connected in parallel to the voltage source of $V=5.0V$.



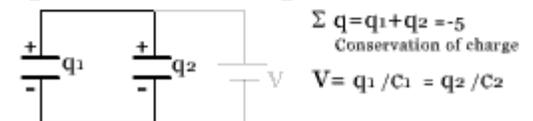
intermediate state

The capacitors are disconnected and then reconnected so that opposite plates face each other.



final state

Charge flows until the two capacitors are again at the same voltage.



The voltage is greyed out to indicate that although no battery is present, the capacitors are at the same potential.

Problem 69:

Electromagnetism \Rightarrow } Ampere Law

Recall Ampere's Law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$.

Since the region one is interested in is a vacuum, one's Ampere Loop encloses all of the current. Thus, the field from each conductor is $B(2\pi r) = \mu_0 I_{enc}$, where, $I_{enc} = I$ and R is the radius of the conductor. (This is a good approximation of the current, as one assumes that the vacuum region in the center is small compared to the area of the conductors.)

Making the approximation that $R \approx d/2$, one has $B = \frac{\mu_0 I}{2\pi R} \approx \frac{\mu_0 I}{4\pi}$. Since both fields contribute in the center, the field is twice that, $\frac{\mu_0 I}{2\pi}$, as in choice (A).

(Also, one can immediately eliminate all but choices (A) and (B) by the right-hand rule. One seeks a +y-direction field.)

YOUR NOTES:

Problem 70:

Electromagnetism \Rightarrow } Larmor Formula

The Larmor formula for power radiated by an accelerated charge is related to the charge and acceleration as $P \propto q^2 a^2$.

The problem gives the following:

A: $q, a \Rightarrow P_A \propto q^2 a^2$

B: $2q, 2a \Rightarrow P_B \propto (2q)^2 (2a)^2 = 16q^2 a^2$

Thus, $P_B/P_A = 16$, as in choice (D).

YOUR NOTES:

Problem 71:

Electromagnetism \Rightarrow Particle Trajectory

One can get a reasonable approximation for the deflection angle as follows.

Assuming that there is no magnetic field, one has from the Lorentz force $F = ma = qE = qV/d$, where one neglects gravitational acceleration. The acceleration is constant, and it is $a = qV/(dm)$.

Recalling the baby physics kinematics equation, $y = \frac{1}{2}at^2 \Rightarrow dy = at dt$ and the fact that $x = vt = at \Rightarrow dx = v dt$ and $t = L/v$, one can calculate the angle as $\tan \theta \approx dy/dx = \frac{at dt}{v dt} = at/v = \frac{qV L}{dmv}$. Take the arctangent to get choice (A).

YOUR NOTES:

Problem 86:

Electromagnetism \Rightarrow Particle Trajectory

There is a force pointing upwards from the Electric field in the y-direction. Suppose the particle is initially moving upwards. Then, the magnetic field would deflect it towards the right... One can apply the Lorentz Force to solve this problem.

If the particle comes in from the left, then the magnetic force would initially deflect it downwards, while the electric force would always force it upwards. Continue applying this analysis to each diagram. It turns out that one has cycloid motion whenever the electric and magnetic fields are perpendicular.

YOUR NOTES:

Problem 87:

Electromagnetism \Rightarrow Faraday Law

From Faraday's Law or Len's Law, one has $\frac{dB}{dt} = -\frac{dI}{dt}$. Since, in order for the balls to move, they must move in a circle, one has $dI = 2\pi d/2$; moreover, the induced magnetic field would point in the opposite direction to the field that was before, and one has a current in a loop from the right-hand-rule. The area of the magnetic flux is just πR^2 , since the field only goes through the cylindrical region of radius R.

Thus, $E(2\pi d/2) = B\pi R^2 \Rightarrow E = \frac{B R^2}{d}$.

Now, recall some mechanics. The torque is related to the moment-arm and force by $\tau = \sum r \times F$, where $F = qE = q \frac{B R^2}{d}$. Since there is a force contribution from each charge, and since, by the right-hand-rule, their cross-products with the moment-arm point in the same direction, one finds the torque to be $\tau = 2(d/2) q \frac{B R^2}{d} = d q \frac{B R^2}{d} = q B R^2$.

Now, recall the relation between angular momentum and torque to be $\sum \tau = \dot{L}$. Replace the $B \rightarrow B$ above to get $L = q B R^2$, and so the system starts rotating with angular momentum as in choice (A). (This approach is due to Matt Krems.)

Note that one can immediately eliminate choice (D) since angular momentum is not conserved from the external torque induced (to wit: electromagnetic induction). Moreover, although choice (E) is true in general, it does not apply to this problem.

YOUR NOTES:

Problem 88:

Electromagnetism \Rightarrow } Ampere Law

Recall Ampere's Law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$, where I_{in} is the current enclosed by the loop dl .

Apply it to the region between a and b , $B(2\pi r) = \mu_0 I \frac{\pi r^2}{\pi R^2} \Rightarrow B = \mu_0 I \frac{r}{2R^2}$, which gives a linearly increasing field, and thus choices (D) and (E) and (A) are eliminated.

Choices (B) and (C) remain.

Apply Ampere's Law to the region outside of the outer sheath. For $r > c$, one has $I_{in} = 0 \Rightarrow \oint \vec{B} \cdot d\vec{l} = 0 \Rightarrow B = 0$. Choice (B) shows the behavior of zero-field outside the sheathed coax cable. Choose that.

YOUR NOTES:

Problem 89:

Electromagnetism \Rightarrow } Trajectory

The only physics involved in this problem is equating the centripetal force with the Lorentz Force, $mv^2/R = qvB$. The rest is math manipulation and throwing out terms of ignorable order.

The radius of curvature used in the centripetal force equation is given by $R^2 = r^2 + (R-r)^2$, and ETS is nice enough to make this geometry fairly obvious in the diagram enclosed with the original question.

Now, note that since $r \ll R$, after expanding the expression for r^2 , one can drop out terms of higher order. Thus, $R^2 = r^2 + (R-r)^2 = r^2 + R^2 + r^2 - 2Rr \approx r^2 + R^2 - 2Rr + O(r^2)$. Canceling the R^2 's on both side, one finds, $r^2 = 2Rr \Rightarrow R = r^2/(2r)$. Plug this into the force equation above to find,

$$mv^2/R = qvB \Rightarrow 2mv/r^2 = qvB \Rightarrow p = mv = qBr^2/2, \langle r \rangle$$

which is choice (D).

YOUR NOTES:

