Solutions for class #10 from Yosumism website

Yosumism website: http://grephysics.yosunism.com

Problem 1:

Electromagnetism⇒}RC Circuit
One can immediately eliminate plots A, C, E, since one would expect an exponential decay behavior for current once the switch is flipped.
More rigorously, the initial circuit with the switch S connected to $a$has the following equation,

$$V-\dot{Q}r-Q/C=0 \Rightarrow \frac{dQ}{dt}=\frac{1}{r}\left(V-Q/C\right)
$$

Once integrated, the equation becomes,

$$Ve^{-\frac{t}{rC}}=V-Q/C \Rightarrow Q/C=V\left(1-e^{-\frac{t}{rC}}\right)
$$

The charge stored on the capacitor after it is fully charged is $Q=CV$ (at $t=0$).
When the switch S is switched to b, the equation becomes,

$$Q/C=\dot{Q}R \Rightarrow Q(t)=Q_0 e^{-\frac{t}{RC}},
$$

where $Q_0=CV$ from the initial connection.
Current is the *negative* time derivative of charge, and thus,

$$I(t)=-\dot{Q}=\frac{Q_0}{RC}Q(t)=\frac{V}{R}Q(t)
$$

The initial current IS $I(0)=V/R$, and thus choice (B) is right.

YOUR NOTES:

Problem 2:

Electromagnetism⇒}Faraday Law

Recall Faraday's Law $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$. Dot both sides with the area $d\vec{A}$. Recalling Stokes' Theorem ($\int\nabla\times\vec{E}\cdot d\vec{A} = -\frac{\partial\vec{B}}{\partial t}\cdot d\vec{A}$), the left side can be converted to the potential, i.e., the emf $\mathcal{E} = -\int\vec{E}\cdot d\vec{l} = -\int\nabla\times\vec{E}\cdot d\vec{A}$.

Finally, from Ohm's Law $V - \mathcal{E} = IR$, one can obtain the current. (Note that $V = 5.0$V is the voltage of the battery. The voltage induced acts to oppose this emf from the battery.)

The problem gives $\frac{dB}{dt} = 150T/s$. The area is just $0.1^2 m^2$. Thus, the induced emf is,

$$\mathcal{E} = \frac{dB}{dt}A = 150/100 = 1.5
$$

Thus, $V - \mathcal{E} = 3.5 = IR \Rightarrow I = 0.35A$, since $R = 10\Omega$.

YOUR NOTES:

Problem 3:

Electromagnetism⇒}Potential

Recall the elementary equations, $V = \int\vec{E}\cdot d\vec{l} = \int\frac{dq}{4\pi\epsilon_0 r}$.

$r = \sqrt{R^2+x^2}$, and $dQ = Q$thus $V = \frac{Q}{4\pi\epsilon_0\sqrt{R^2+x^2}}$, as in choice (B).

YOUR NOTES:

Problem 4:

Electromagnetism$\Rightarrow$}Small Oscillations

The potential is determined in the previous problem to be $V = \frac{Q}{4\pi\epsilon_0\sqrt{R^2+x^2}}$. The field is given by $\vec{E} = -\nabla V$. Before taking derivatives, one can simplify the potential since it is given that $R >> x$.

Binomial expand it ($(1+y)^n \approx 1+ny$, for $y$small) to get

$V \approx \frac{Q}{4\pi\epsilon_0 R}(1+\frac{x^2}{2R^2})$

Taking the derivative, using the equations $\vec{E} = -\nabla V$, and $\vec{F} = q\vec{E}$, one gets,

$-q\frac{Q}{4\pi\epsilon_0 R}\frac{x}{R^2} = F = m\ddot{x}$.

Small oscillations have the same form as simple harmonic oscillations, i.e., $\ddot{x} = -\omega^2 x$. The angular frequency is $\omega = \sqrt{q\frac{Q}{4\pi\epsilon_0 R^3 m}}$, as in choice (A).

YOUR NOTES:

Problem 19:

Electromagnetism⇒}Coulomb's Law

The particle obeys a Coulomb's Law potential, $V = \frac{kZ_1Z_2q^2}{r}$. In this case particle 1 is a Helium atom, which has charge $Z_1 = 2$, while particle 2 is silver, with $Z_2 = 50$. Thus,

$$V = \frac{100kq^2}{r\,\cdot} < br/ >$$

Conservation of energy requires that when the incident particle is at its closest approach, $5MeV = \frac{100kq^2}{r}$. Recall that $k = 9E9$, $q = 1.602E{-}19$, convert everything to $SI$to get $r \approx 2.9E{-}14$

YOUR NOTES:

Problem 24:

Electromagnetism⇒}Conductors

This problem involves applying Coulomb's Law $F \propto q_A q_B / r^2_{AB}$ to conductors. The charge travels from conductor to conductor and equilibriates instantaneously due to the requirement that two touching conductors must be at an equipotential. This means that if conductors 1 and 2 touch then their potentials are related by $V_1 = V_2$. Because the problem involves spherical conductors, the potential has the form $V \propto q_A q_B / r_{AB}$.

The initial force between the two conductors is F, where $q_A = q_B = Q$.

After C is touched to A, the charge becomes $q_A = Q/2 = q_C$, since each conductor shares the same charge out of a total of Q (to wit: each has half of the total charge).

When C is touched to B, the charge becomes $q_C = 3/4 Q = q_B$, since each conductor shares the same charge out of a total of $Q + Q/2$ (to wit: $\frac{1}{2} 3/2 Q = 3/4 Q$ for each conductor).

When C is removed, one calculates the force from Coulomb's law and the final charges on A and B determined above to be, $F = 3/8 Q^2 / r^2_{AB} = 3/8 F$, as in choice (D).

YOUR NOTES:

Problem 25:

Electromagnetism⇒}Capacitors

Recall the following truths (held to be self-evident?) on the subject of capacitors: 1. series capacitors have equal charge (Equivalent capacitance of two capacitors is $1/C_{eq}=1/C_1+1/C_2$); 2. parallel capacitance have equal voltage ($C_{eq}=C_1+C_2$); 3. $Q=CV$; 4. $U=\frac{1}{2}CV^2$.

(A) Initially, before the switch is closed, only C_1 has a voltage across it, and hence it is charged. $Q_0=CV$. But, afterwards, since the voltage stays the same, one has $Q_1=Q_2=CV$; hence, $Q_0=\frac{1}{2}(Q_1+Q_2)$.

(B) $V_1=V_2 \Rightarrow Q_1/C_1=Q_2/C_2$. Since $C_1=C_2$, one has $Q_1=Q_2$. This is true.

(C) By definition of circuit elements in parallel, one has each capacitor at the same potential. This is trivially true. $V_1=V_2=V$

(D) Since one determined from (C) that the capacitors are at the same voltage, then because they have the same capacitance, they have the same energy as per $U=\frac{1}{2}CV^2$. True.

(E) This is false, since $U_0 = \frac{1}{2}CV^2$, initially. In the final state, *each* capacitor has energy $\frac{1}{2}CV^2$. The sum of energies is thus $2U_0$.

YOUR NOTES:

Problem 26:

Electromagnetism⇒}Resonance Frequency

One wants to tune one's radio to the resonance frequency (a.k.a. the frequency at which impedance is matched). The resonance frequency of an LRC circuit is given by $\omega^2 = 1/LC$, where the quantities involved are angular frequency, inductance, and capacitance. Solving for C, one has $C = 1/(L\omega^2) \approx 1/(2E-6*36*100E6) = 1/(7.2E-11) \approx 0.1E-11 = 1E-12$. This is choice (C). The hardest part of his problem, of course, is doing the math without a calculator. Easy.

YOUR NOTES:

Problem 43:

Electromagnetism⇒}Stokes Theorem

Recall Stokes' Theorem $\oint \nabla \times u d\vec{a} = \int \vec{u} \cdot d\vec{l}$. The left side of the equality is easier to evaluate, so evaluating that, one has $\nabla \times u = 2$. The area is πR^2, and thus $\int \vec{u} \cdot d\vec{l} = 2\pi R^2$.

YOUR NOTES:

Problem 46:

Electromagnetism⇒}Faraday Law

Recall Faraday Law, $\epsilon = -\frac{d\Phi}{dt}$, where $\Phi = B \cdot dA$. Since the magnetic field is constant, the equation simplifies to $\epsilon = -B \cdot \frac{dA}{dt}$ for this case.

$B \cdot A = B\cos(\omega t)\pi r^2$, and thus $d\Phi/dt = -\omega \sin(\omega t)\pi r^2 = -\epsilon = -\epsilon_0 \sin(\omega t)$. Solving for angular momentum, one has $\omega = \epsilon_0/(B\pi R^2)$.

Alternatively, one has $A = \pi r r(t) \Rightarrow \dot{A} = \pi r \dot{r} = \pi r v$. Since $v = \omega r \sin(\omega t)$, one has $A = \pi r \omega r \sin(\omega t)$. Plug it into Faraday Law and solve for angular velocity.

YOUR NOTES:

Problem 47:

Electromagnetism⇒}Faraday Law

Recall Faraday's Law, $\epsilon = \vec{E} \cdot d\vec{l} = -d\Phi/dt$, where $\Phi = \vec{B} \cdot d\vec{A}$. In words, this means that a changing flux (either a varying field or radius) induces a voltage.

The field is given as just B. The area of the loop is just πR^2, i.e., the cross-sectional area of the cylinder. As the cylinder is spun around, its flux changes at the rate of N rps. The change in flux is thus $NB\pi R^2$, and this is the magnitude of the potential difference in choice (C).

(Also, one can drop out the other choices from units. And, since the cylinder is moving in a magnetic field, the non-zero flux demands a voltage, so (A) can't be it.)

YOUR NOTES:

Problem 49:

Electromagnetism⇒}Relativistic Fields

For motion in the x direction, one has the following equations for the E and B fields,

Ex = E'x

 Bx = B'x

Ey = ?(E'y – vB'z)

By = ?(B'y – v/c^2 E'z)

Ez = ?(E'z – vB'y)

Bz = ?(B'z – v/c^2 E'y)

Since $E'_z = \sigma/(2\epsilon_0)$(with all other primed components 0), the transformed field is just $E_z = \gamma E'_z$, as in choice (C).

(Recall that $\gamma = 1/\sqrt{1-\beta^2}$, where $\beta = v/c$)

If one forgets the Lorentz-transformed fields, one can also quickly derive the answer for this case. Since the transformed charge density is Lorentz contracted in one of its area dimensions, one has $\sigma = \gamma\sigma'$. One can tell by symmetry of the surface that the other field components cancel, and one again arrives at the result for E_z as above.

YOUR NOTES: